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# The development and analysis of a cooperative decision model for product pricing and financial structure

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The development and analysis of a cooperative decision  
model for product pricing and financial structure

by

John Jay VanSickle

A Dissertation Submitted to the  
Graduate Faculty in Partial Fulfillment of the  
Requirements for the Degree of  
DOCTOR OF PHILOSOPHY

Major: Economics

Approved:

Signatures have been redacted for privacy.

Iowa State University  
Ames, Iowa

1980

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## CHAPTER I. INTRODUCTION

Grain production and marketing are of vital importance to Iowa's and the nation's economy. Iowa ranks as one of the nation's leading producers of corn and soybeans. In 1978, Iowa's production accounted for 21 percent of the nation's 7.08 billion bushel corn crop and 15.6 percent of the nation's 1.84 billion bushel soybean crop.

The marketing of these grains plays an important role in agriculture. The Warehouse Division of the Iowa Commerce Commission reports that as of January 15, 1979, approximately 850 firms were legally licensed to store grain in Iowa. These firms had a total storage capacity of more than 400 million bushels. In Iowa, as in many grain marketing states, the majority of the firms licensed to store grain are cooperatives.

## Problem Statement

A cooperative association is an organization of firms which is controlled by those who use it and is operated for their mutual benefit as patrons. The cooperative association represents an extension of each member firm's operation in which the member firms operate the cooperative jointly to improve their separate operations.

A cooperative association differs in its operation from a proprietary corporate enterprise because of its organizational structure and principles of operation. In a cooperative, the financial benefits flow to the patrons on the basis of patronage, while in a corporate enterprise the financial benefits flow to the owners on the basis of investment in the enterprise.



Another distinguishing characteristic of cooperatives is the basic principle that ownership and control is in the hands of the member patrons who use the services of the cooperative. This differs from corporate enterprises where ownership and control is based on investment.

Cooperative associations also differ from corporate enterprises because the former operates on a cost basis rather than for a profit for investors. Any revenues realized over and above expenses (net savings) belong to the member firms who patronize the cooperative. Net savings are often realized because costs cannot be estimated accurately in advance. The patrons normally pay the competitive price for the goods or services they obtain from the cooperative, then receive adjustments at the end of the cooperative's fiscal year when costs are known. The adjustment goes back to the member-patrons as refunds based on patronage. This refund allocation thus allows the cooperative to operate at cost.

Another distinguishing characteristic of cooperative associations is the limited return on capital. The return on capital is the financial benefit or profit given for investing capital in the firm. In proprietary corporate enterprises the return on capital is the primary purpose for investing in the firm. In cooperatives, the members are primarily interested in the benefits they derive as patrons. These benefits may be either financial or in the form of services performed. The returns are distributed on a basis of patronage instead of on a basis of capital investment. Because the returns are allocated in this manner, the return based on capital investment is either very little or nonexistent.

Another characteristic of cooperatives is the obligation of member firms to provide the needed capital. Since returns based on capital investment are limited, there is no profit motive for providing capital as investors. The member-patrons therefore assume the responsibility of providing the required capital.

Cooperatives also differ from corporate enterprises in the instruments available for providing the required capital. Corporate enterprises use primarily debt, common stock, and preferred stock. Cooperative associations have all these instruments plus that of deferred patronage dividends. Patronage dividends are the excess of revenues over costs experienced in operating the cooperative. The dividends are paid on a basis of a percentage of the patronage dividends paid in cash at the end of the fiscal year and the remainder paid in cash in a later period. The portion paid in cash in a later period is the deferred patronage dividend. The percentage of patronage dividends held for deferred payments and the periods and manner in which they are held may vary among cooperatives. These deferred dividends are used by the cooperative and are a form of member investment in the cooperative.

Cooperatives also differ from corporate enterprises in the treatment of common stock. In the corporate enterprise, the price of common stock is determined in the market place and is dependent on the earnings of the firm and the capitalization rate associated with the firm. Cooperative association common stock is almost always fixed in price in the articles of incorporation of the firm and does not vary with the earnings of the association. The determination of the optimal capital structure in

corporate enterprises is most commonly done by maximizing the value of each share of common stock in the firm. Since cooperatives are assumed to operate at cost and have a common stock price fixed by decision-makers, a different maximand is needed in which the common stock price is a choice variable.

The distinguishing characteristics of cooperatives create a need for a difference in the analysis of pricing and production policies. The cooperative enterprise has not had the extensive study which corporate enterprises have had in this area. Hardie (25), Helmlberger and Hoos (26), Ladd (33), and Royer (42) have all developed differing optimizing models for cooperative pricing and production policies. These models have the objective of maximizing either the price paid to the member-patron, total sales, or member profits. The unique operation of cooperatives in any of these methods dictates the use of special treatment differing from that of corporate enterprises.

The distinguishing characteristics of cooperatives also create a need for the analysis of cooperative financial structure to differ from the analysis of financial structure in corporate firms. The difference in the treatment of common stock, the addition of deferred patronage dividends as an instrument for capital financing, and the principle of limited return on capital all indicate the need for specification and analysis of an appropriate objective for analyzing cooperative financial structure which will differ from the objective of the corporate firm.

### Prior Research

Research dealing with the topic of this study has been scarce. A number of studies have focused on the economies of size and spatial distribution of grain elevators. In 1969, Halverson (24) studied the economies of scale in Iowa's country elevators. He developed an engineering cost equation for in-plant operations and combined this with a derived assembly cost equation to determine the economies of scale. Mikes (35) conducted a similar study in 1971 in which he developed a forecast for grain production in 1980 and combined this with his study on economies of scale to determine the structural adjustments he felt should take place in Iowa's grain industry. Other studies have used similar methodology in analyzing the feed (36) and fertilizer (40) industries.

Another area of research has focused on optimizing models for pricing and production in cooperatives. Early literature on cooperatives used the idea that the central economic units for analysis were the member firms. Emelianoff (17) viewed the cooperative association as an organization of economic units, each having their own economic independence while coordinating their business activities through the cooperative owned and controlled by them. An important element of this earlier work was the absence of a decision-maker in the cooperative. Robotka (41), Phillips (38), Aresvik (5), and Trifon (47) all embraced the idea of no decision-maker in the cooperative.

Because of a lack of a cooperative decision-maker, the objectives of the cooperative received little attention. The idea that cooperatives

operated for the benefit of its member-patrons was as far as the definition of the objective went. Clark (11) developed a model for the cooperative to minimize the cost of operation. This was a useful formulation because one of the founding principles of cooperation was to give service at cost. With the Clark model, the cooperative supplies goods and services to the members and markets their output. The cooperative operates at the point where minimum average cost is attained. This model was criticized by Aizsilnieks (2), who argued that the cooperative does not have an independent output decision because it must market what the member firms demand of it.

Savage (44) was one of the first to challenge the nonexistence of a decision-maker in the cooperative. He argued that failing to recognize the cooperative as a "firm" ignored the conception of the cooperative which farmers and cooperative leaders had. Enke (18) was one of the early researchers to embrace the cooperative decision-maker idea. His model was of a consumer cooperative association in which the decision-maker makes decisions concerning the operation of the cooperative. He argued that the cooperative decision-maker should have the objective of maximizing the members net consumer surplus. With this the cooperative decision-maker would set the product price equal to the marginal cost of producing it.

Helmberger and Hoos (26) embraced the cooperative decision-maker idea in a model of a single product firm. This model provided a theoretical basis for maximizing the price paid to the member firms in a single product marketing cooperative. The framework of the model

implies that when net savings are maximized the price paid to the member firm for the product is maximized. Hardie (25) extended this model to the multiproduct cooperative. He formulated a linear programming model and argued that the shadow prices he derived from it should determine the method for distributing cooperative net savings. Ladd (33) used the arguments of the Helmberger and Hoos model to extend into a different multiproduct cooperative model. His model contained three services - the sale of an item used as a productive input by members and nonmembers, an excludable public good provided only to members, and a bargaining service benefiting both members and nonmembers by affecting the price they receive for their raw product. Ladd considered the alternative objectives of maximizing the raw material price received by members and the quantity of raw material marketed through the cooperative. Ladd showed that optimizing conditions for these two objective functions differed substantially from one another and from those of a profit maximizing firm.

Royer (42) incorporated the idea of central economic units into his general nonlinear model for cooperatives. He did this by distinguishing members and nonmembers who may do business with the cooperative. He derived supply functions for products that the member and nonmember patrons sell to the cooperative and demand functions for the products the member and nonmember patrons purchase from the cooperative. He defined the objective of the cooperative as the maximization of the member profits. Royer derived the Kuhn-Tucker conditions from his model and interpreted the economic implications of the Kuhn-Tucker

conditions.

Eversull (20) demonstrated how a practical application of Royer's model could be used as a planning guide by cooperative decision-makers. He did this by specifying a model for a hypothetical cooperative association and then derived and discussed the results obtained from the model.

Research dealing with optimization of the capital structure in corporate enterprises is typified in the analysis done by Vickers (57). Vickers looked at two sources of capital, debt and owners' equity. He developed an equity cost function which gives the capitalization rate of owners' equity as a function of the coefficient of variation in the firms total net operating stream, the total capital employed, and the leverage ratio (debt/total liabilities). He then developed a debt cost function which gave the average interest rate of debt as a function of the coefficient of variation of the earnings stream available to cover the interest on debt and the leverage ratio. Vickers then selected the financial leverage that maximized the rate of return on the book value of owner investment. From this he found that the optimum degree of financial leverage was that at which the marginal rate of return on equity is equal to the marginal rate of interest on debt. For the more general case, he said that the optimum allocation of the firm's demand for capital over alternative capital sources would be such as to equate the cost of each capital source at the margin.

Other sources of research on capital structure have dealt more specifically with cooperatives. Tubbs (48) measured the effects on

member profits, liquidity, and asset accumulation under differing financing plans employed by the cooperative. His computer simulation model showed that sizeable capital requirements of members by the cooperative can reduce substantially the profits and assets accumulated by the members over time. In selecting an optimal financing plan, Tubbs minimized the financial sacrifices of the investing member as a basis for determining the optimum.

Other cooperative studies have developed models for optimizing the capital structure and then compared actual practice with the results obtained in the model. Snider and Koller (46) conducted a study on Minnesota dairy marketing cooperatives. They found that the average cost of capital could be lowered simply by increasing the amount of debt relative to other forms of financing in the cooperative. This study found that the cost of debt was considerably less than the cost of deferred patronage dividends, which was defined as equivalent to the member's opportunity cost of capital.

Similar studies in other states have resulted in similar conclusions, i.e., too much in deferred patronage dividends was being used and not enough debt was used as capital. Nervik and Gunderson (37) found in their study of South Dakota cooperatives that too heavy a reliance on deferred patronage dividends can cause an inadequate permanent capital base. Deferred patronage dividends are only a semi-permanent source of capital since they must be paid out in cash at a later date. This creates problems in obtaining borrowed capital which, in turn, creates the problem of not enough capital provided by debt.



Findings similar to these were obtained by Korzan and Gray (32) in an Oregon study. They studied data of capital structures in Oregon cooperatives and compared these to alternative financial plans. They concluded that cooperatives have access to borrowed funds at cheaper rates than do the individual member producers. They concluded a net gain would result if cooperatives borrowed operating capital and returned more equity capital back to the producers.

Wilson (59) came up with another conclusion from his financial study on cooperatives. He recommended that equity capital be serviced first, and then patronage dividends be declared. He argued that a return on member capital should be equal to a moderate, fixed rate, serviced like interest is on debt. Current law allows such an allocation procedure,<sup>1</sup> however, Wilson asserts that many would argue that this is not a real cost and would violate the cooperative principle of operation at cost.

There have been a number of descriptive and applied studies completed, but relatively few have dealt with the theoretical framework in the financing of cooperatives. Fenwick (21) did, however, develop a theoretical framework for minimizing the cost of capital in a cooperative. He developed equations for determining the current value of a member's investment, and then constructed a function identifying the value of the cooperative to the members and an average cost of capital function. From this he found that as the amount of borrowed funds increases, the average cost of capital decreases. Also, as the revolving period decreases in length, the member's opportunity cost

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<sup>1</sup>Code of Iowa (12, Section 30).

decreases. Dahl (15) conducted a study on Wisconsin cooperatives which also utilized the concept of minimizing the cost of capital. His study yielded the standard conclusion that not enough debt was being used as a financing tool.

Coffman (14) developed a linear programming model for analyzing the annual financial condition of a local multi-enterprise cooperative. The model was used for long range financial planning within the cooperative and used to analyze the effects of different financing policies on different classes of member patrons. Beierlein and Schrader (8) developed a deterministic simulator of cooperative and patron cash flows and used it to evaluate the impact of changes in cooperative capital structure on the level of patron benefits. They reach inconclusive results because of the interaction of particular capital plans with patron characteristics.

#### Summary of Prior Research

The studies reviewed concerning cooperatives have concentrated on the problem of either developing the theory for cooperative pricing and production policy in cooperatives or determining the optimal capital structure within the cooperative. No decision model could be found which might coordinate these two problems within the cooperative.

In the cooperative pricing and production problem, much was identified which dealt with the theory but the empirical testing of the models developed was relatively little or nonexistent. Royer seems to have drawn together the ideas from earlier authors to develop his

general linear programming model for cooperative production and pricing policy.

The study of cooperative financing has been much more empirical than that of cooperative pricing and production. Several authors (19, 31, 36, 45, 48, 58) were able to draw conclusions about present cooperative financing practices, the most common conclusion being that not enough debt and too much deferred patronage dividends was used in financing the cooperative.

The theory concerning the optimal capital structure in cooperatives has not had the extensive research that cooperative pricing and production has. The most common approach to evaluating cooperative financing has been the development of linear programming models and simulation models. The most basic theoretical approach was developed by Fenwick in minimizing the cost of capital used for financing the cooperative. The other source of theory reviewed concerned the financing of corporate enterprises developed by Vickers. His approach to optimal capital structure was to maximize the rate of return on the book value of owners' investment for increases in financial leverage. No adaptation of this theory to cooperatives was found.

### Objectives

The review of literature points to several problems associated with analyzing cooperatives. First, there has been no study which has effectively described the cooperative decision nexus. Many studies have

dealt with either the production and pricing problem or the financing problem, but never both. Each author has assumed the decisions to be independent enough in nature to analyze separately.

Second, the literature concerning the production and pricing policies within cooperatives has been primarily theoretical. No empirical testing of the models described has taken place. Royer developed a general linear programming model for cooperatives and interpreted the Kuhn-Tucker conditions he derives from it. Eversull applied the model to a hypothetical cooperative to show how the model might be used as a planning guide for cooperative decision-makers. Beyond this study no practical application of the model has been done.

Third, even though much literature has dealt with the financing problems within cooperatives, few models have been developed for a theoretical analysis. Many linear programming models and simulation models were developed which were based on the cash flows to member patrons, but minimizing the cost of capital has been the most basic theoretical approach used. A cooperative association has been defined as an organization of firms operated jointly to improve each member firm's operation. It has not been adequately shown that minimizing the cost of capital is a rule followed by cooperatives nor has it been shown that this is the objective most consistent with cooperative principles.

In light of the problems pointed out, an attempt will be made here to define a more complete and accurate description of the cooperative decision nexus. The relationship of the production and pricing decision

and financing decision will be shown.

Once the decision nexus has been defined more accurately, theoretical models will be developed which this study will use as a basis for evaluation of actual cooperative practices. Data collected from local farmer cooperatives in Iowa will be used for empirical estimation of the production and pricing model. The treatment of the finance model will only evaluate the economic implications of the Kuhn-Tucker conditions of the model. No empirical estimation of the finance model will be done.

#### Following Chapters

In Chapter II, the decision nexus of the cooperative is presented. The relationship of the production and pricing decision and financing decision is shown.

Chapter III takes a close look at the theoretical model concerning the production and pricing decision. Chapter IV specifies the empirical procedures used in estimating the production and pricing model and Chapter V discusses the results obtained from the empirical analysis.

Chapter VI presents the theoretical model concerning the finance decision. Chapter VII discusses the economic implications of the Kuhn-Tucker conditions derived from the general finance model and models with common-stock price and capital-employed restrictions. Chapter VIII consists of a Summary, Conclusions, and Suggestions for Further Research.

## CHAPTER II. THE COOPERATIVE ASSOCIATION

## DECISION NEXUS

The objective of the cooperative decision-maker in the short-run production and pricing model developed by Royer and discussed in the introduction was the maximization of the total collective profits of the member patrons in the cooperative association. The logic for the derivation of this objective was that, if the cooperative is viewed as an extension of the member patrons or if it is understood that the purpose of the cooperative is to benefit the member patrons, maximization of the total profits of the members patrons is consistent with the assumption that member patrons maximize profits.<sup>1</sup> There exists no generally accepted objective for the financial decision facing the cooperative decision-maker. Fenwick (21) suggested that minimizing the cost of capital could be one method of obtaining a defined capital structure. Other authors have developed simulation models which maximize the present value of cash flows to the member patrons of the cooperative association. Vickers<sup>2</sup> suggested that the financial function of the corporate firm cannot be understood or discharged in isolation from a larger nexus of forces. These forces, when taken together, determine an optimum economic position of the corporate firm.

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<sup>1</sup>J. S. Royer (42, pp. 59-60).

<sup>2</sup>D. Vickers (57, p. 37).

The decisions facing the cooperative decision-maker are in fact dependent upon a larger nexus of forces involving all phases of the cooperative economic structure. These decisions must be analyzed for their interdependence to determine the forces which jointly determine the optimum economic structure of the cooperative association.

In this chapter the operating and planning decision problems of the cooperative association which lead to the optimum economic structure will be looked at relative to the "sequential decision-making points" at which the enterprise is examined for optimum economic structure. The following chapters will deal more specifically with the independent decisions of the cooperative decision-maker.

#### The Cooperative Association

A cooperative association is an organization of firms joined together for the purpose of processing and marketing their products or supplying the inputs they use in production. The activities may or may not be limited to members, in fact, many cooperatives do business with patrons who are not members.

Figure 2.1 shows the relationships between the various groups of the cooperative association. The cooperative purchases unprocessed products (set X) from member and nonmember patrons and supplies them with variable inputs (set Y) which they use in production. The cooperative determines the price it will pay for purchased unprocessed products and the price it charges for the sale of variable inputs.

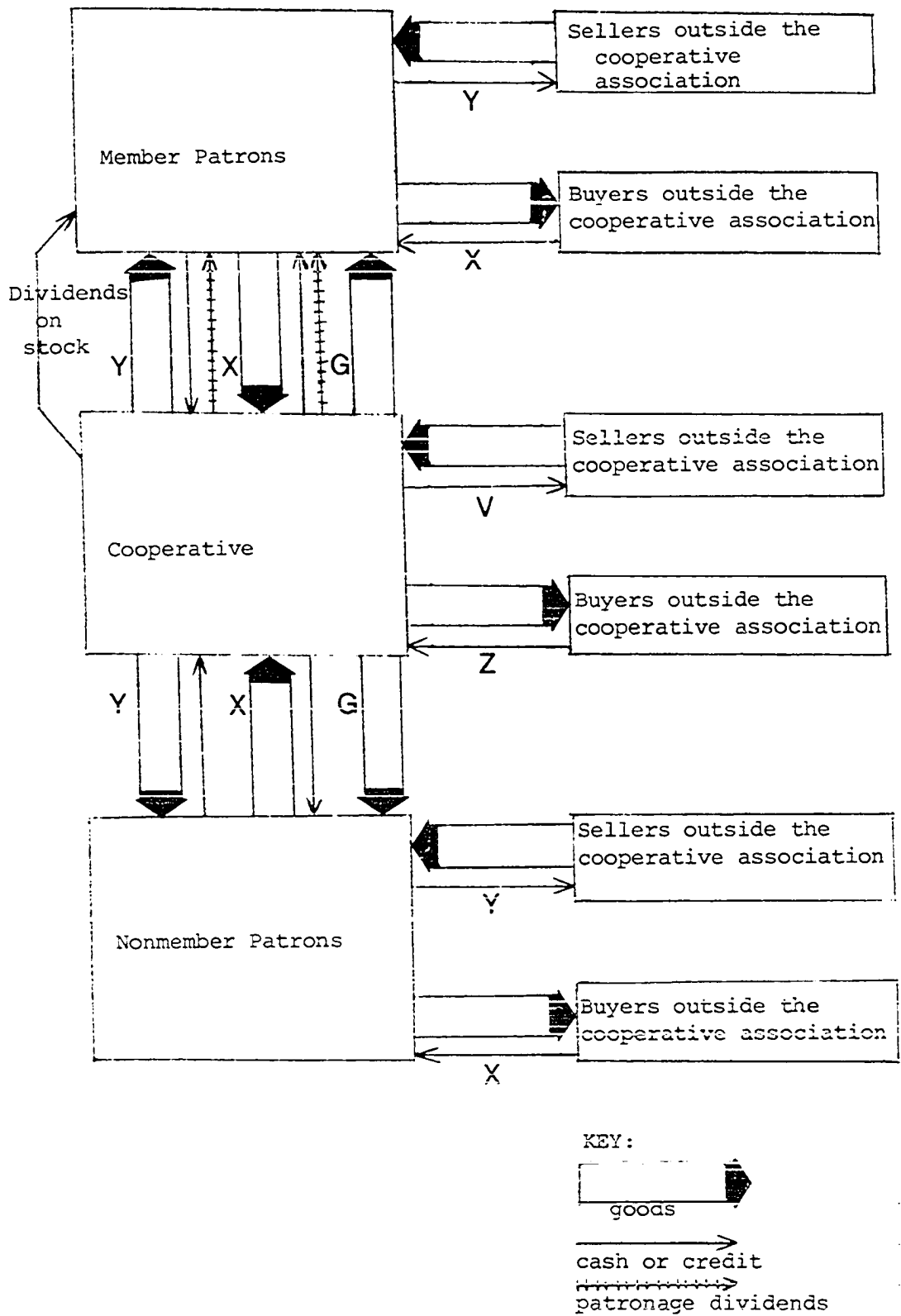


Figure 2.1. Model of the cooperative association (adapted from Royer, Fig. 2.1, p. 29)



Some inputs are not sold (set G) because they are public goods which cannot be excluded from nonpayers. Instead, the cooperative provides them free of charge. The cooperative also purchases variable inputs (set V) from sellers outside the cooperative association and sells finished products (set Z) to buyers outside the cooperative association. In this model, the member patrons receive patronage dividends from the cooperative on the basis of patronage with the cooperative. This is one method of distributing a portion of the net savings. Despite the principle of operation at cost<sup>1</sup> cooperatives do often realize a net savings in the operation. Net savings occur in the operation of the cooperative for two basic reasons. First, cooperatives cannot accurately forecast what the true cost of operation will be. The cooperative may charge a price to its members for a service which exceeds the true cost of the service, therefore generating a net savings in the operation. Secondly, the cooperative may choose to have net savings in its operation to pay for upkeep and expansion of its facilities. The cooperative will generate the net savings and retain a portion of them via deferred patronage dividends or retained savings to accommodate this need.

An alternative method of allocating net savings is with dividends on stock, primarily to pay a limited return on investment in the firm. Dividends cannot be paid on any type of investment other than common stock investment, otherwise the cooperative loses the tax benefits derived from the Internal Revenue Service which allows savings allocated by specific rules to be nontaxable.

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<sup>1</sup>Martin A. Abrahamson (1, p. 50).

Also observable from the model is that members and nonmembers are not required to do business exclusively with the cooperative association. Both groups may do business outside the cooperative association for the variable inputs (set Y) and unprocessed products (set X).

### The Cooperative Financial Statements

The process of determining the optimum economic structure of the cooperative requires an understanding of the financial statements. These financial statements include the balance sheet, which describes in money value terms the position of the firm as of a specified date, and the income statement, which summarizes the flows of revenues and costs during a preceding period of time. The most common period of time involves a calendar year, but may involve different lengths of time.

The balance sheet and income statement for the cooperative is quite similar to that found in the corporate enterprise. Figure 2.2 indicates the form of the financial statements for the cooperative.

Consider the balance sheet first. Double-entry bookkeeping ensures that total assets must be the same as total liabilities. The assets side of the balance sheet for the cooperative is similar to that for a corporate enterprise. The total assets indicate the total investment that has been made in the cooperative. The asset accounts describe the various ways in which the money capital has been invested in the cooperative, in essence, the structure of the investment in the cooperative. The money capital may be invested in different activities. The

BALANCE SHEET

<u>Assets:</u>		<u>Liabilities:</u>	
<u>Current assets</u>		<u>Current liabilities</u>	
Cash	xxx	Accounts payable	xxx
Accounts receivable	xxx	Short-term debt	xxx
Inventory	xxx	<u>Long-term capital sources</u>	
<u>Fixed assets</u>	xxx	Debt	xxx
		Preferred stock	xxx
		Owner equity	
		Common stock	xxx
		Earned surplus	xxx
		Deferred patronage refunds	<u>xxx</u>
TOTAL ASSETS	<u>xxx</u>	TOTAL LIABILITIES	<u>xxx</u>

INCOME STATEMENT

Total sales	xxx
Less: Variable factor costs	xxx
Fixed factor costs	<u>xxx</u>
TOTAL operating costs	<u>xxx</u>
Net operating income	xxx
Less: Interest on debt capital	<u>xxx</u>
NET savings	xxx

Figure 2.2. Cooperative financial statements

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Less: Preferred stock dividends	xxx
Common stock dividends	xxx
Patronage refunds	<u>xxx</u>
 TOTAL allocations	 <u>xxx</u>
Retained earnings before tax	xxx
Income tax liability	<u>xxx</u>
Retained earnings	<u>xxx</u>

Figure 2.2 (Continued)

assets show the combinations of assets which have been selected and employed by the firm.

The liabilities side of the balance sheet indicates the total money capital employed in the firm. The liabilities side of the balance sheet for the cooperative is similar to that of a corporate enterprise with the exception of owner equity. The owner equity is the total capital which has been supplied by the owners or, in the cooperative, member patrons. In a cooperative, member patrons supply capital via common stock and earned-surplus, both commonly used by corporate enterprises. Cooperatives, however, also obtain capital from member patrons with deferred patronage dividends. Patronage dividends are exclusive to cooperatives as a method of allocating net savings. The deferred portion of these patronage dividends gives the corporation an additional instrument for obtaining capital.

The principal point to note in the balance sheet is that the liabilities side describes the various sources from which money capital has been obtained, and the assets describe the uses to which the money capital has been employed. Thus, the balance sheet shows the financial structure of the firm, the structure of its investments and the structure of its financing methods.

The income statement indicates the revenues and costs realized during the preceding financial period. Net operating income equals income minus all costs except the interest paid on debt capital, and net savings equals net operating income minus interest on debt. The major difference in the cooperative from the corporate firm is in the total allocations and handling of the income tax liability. In the corporate firm income taxes are determined as a function of the net saving while in a qualified 521 cooperative, income taxes are based primarily on the amount of retained earnings. The allocations of net savings also differ because patronage dividends are allocated to members in cooperatives and not in corporations.

#### The Management Functions

The task of cooperative management encompasses all of the aspects which affect the cooperative financial statements. A cooperative association differs from a corporate enterprise in that the former is organized and operated for the benefit of its member patrons. The operation for the benefit of the member patrons entails a different

set of objectives by which the cooperative must be operated. As was pointed out earlier, the objective of maximizing the profits of the firm is not the objective assumed in most cooperative studies. This study will assume that the objective of the cooperative is that of maximizing the total collective profits of the member patrons.

In fulfilling the objective of maximizing the total collective profits of the member patrons, the cooperative decision-maker is faced with a set of enterprise management decisions. These management decisions can be categorized by the levels of economic significance associated with each task. The first task which faces the cooperative decision-maker is the task of determining the asset and capital structure of the firm. Second, there is the task of the short-run functioning and management of the cooperative in accordance with the structure determined in the first decision.

The task of determining the structure of the cooperative organization involves decisions which determine the size and structure of activities which best serves the objective of the member patrons. In determining the structure, the cooperative decision-maker is concerned with the combination of activities which best serve the member patrons, which may involve the decision of adding new activities to the enterprise, or changing or deleting existing activities. The determination of structure can be divided into two separate and distinct decisions. First, the cooperative decision-maker must determine the level of capital investment to have in the cooperative and the combination of liabilities necessary for this capital. Second, the cooperative

decision-maker must determine the investment portfolio by which to use this capital. This involves the decision of choosing the amount and combination of investments in assets necessary to maintain the size, structure, and operating processes of the cooperative.

Finally, the cooperative decision-maker must be concerned with the short-run functioning of the firm. In the cooperative, this involves the determination of the level of production and the price paid or received for products handled by the cooperative association. The management of the cooperative in the short-run is reflected in the income statement. Determination of the production and pricing policies affects the revenues and costs of operation, the central points of the income statement.

#### The Decision Nexus

The analysis of the cooperative management decision process can be analyzed in light of the different decision tasks and their proximity in the successive periodic planning dates. The perspective which shall be adapted for the analysis of the cooperative decision process is that the structure decision of the cooperative is logically, as well as temporally, made prior to the short-run operating decisions of the cooperative decision-maker. The financial structure will be determined in conjunction with the asset structure. To determine the structure in this way, it is assumed that a set of asset investment opportunities are given and the returns on these investments are known with certainty. Given this set of investment opportunities, we can

specify the function which describes returns to capital investment. This function will be used in determining the financial structure of the cooperative. The financial structure determined in this way specifies the total capital to be employed by the cooperative and the asset portfolio. The interdependence of the asset and financial structure decisions cause the two decisions to be made simultaneously.

In conceiving the cooperative decision problem, we can divide the decision problem into three interdependent components: one, the production and pricing problem; two, the investment problem; and three, the financing problem. The solution to the production and pricing problem is the short-run decision problem and will be reflected in the revenue flow in the income statement and in the structure of the operating costs. The solution to the investment problem will be reflected in the structure of the assets side of the balance sheet. The solution to the financing problem will be reflected primarily in the liabilities side of the balance sheet. The order of the decisions made can be reflected in the planning dates under which they are solved. The financial and asset structure problems are long-run decision problems and must be made prior to the production and pricing decision. Once the level of capital employed has been chosen in the long-run financing problem, the cooperative decision-maker has simultaneously determined the combination of assets or activities to employ. The determination of the production and pricing problem is a short-run decision which is made with the established structure determined in the financing and investment problems.



In the remainder of this text, we shall be looking more closely at the short-run production and pricing problem and the long-run financing problem in the cooperative. We shall look at the investment problem only from the standpoint of the prospective member patron in determining the value of membership in the cooperative.

The determination of the decision problem outlined previously would suggest that the financing problem should be analyzed first since it is made prior to the production and pricing problem. The text here, however, will first look at the short-run production and pricing problem. This is done because of the previous work relative to this decision problem. A discussion of the theory involved in this problem is presented and an empirical analysis of the results is presented. The financing problem of the cooperative is then analyzed. The financing problem is presented last since it involves the development of new theory for the analysis.

### CHAPTER III. COOPERATIVE PRICING AND PRODUCTION SUB-MODEL

Various studies have dealt with the development of optimizing models for cooperatives in connection with production, pricing, or both. Various objective functions have been specified, each giving different results. The most recent and complete was the model developed by Royer. His general nonlinear model specified the objective function as being the maximization of member profits. He derived the Kuhn-Tucker conditions and interpreted their economic implications.

The sub-model developed in this chapter will use many of the ideas developed by Royer. The same objective function will be utilized and minor modifications will be made in the structure.

#### The Cooperative Association in the Short-Run

When dealing with the short-run pricing and production model, we are concerned with the relationship of the members and nonmembers to the cooperative association. Figure 2.1 shows the relationship which exists between these various groups. The short-run pricing and production decision involves the determination of the optimum level of production and the optimum price for products handled by the cooperative association. By definition of the model depicted in Figure 2.1, the cooperative association purchases unprocessed products (set  $X$ ), sells variable inputs (set  $Y$ ), and supplies public goods (set  $G$ ) to the member

and nonmember patrons. The member patrons receive patronage dividends from the net savings generated by the cooperative and dividends on common stock. A final important characteristic of the model is that members and nonmembers are not bound into doing business with the cooperative association. Both groups may do business outside the cooperative association.

In the pricing and production model we can assume the dividend on common stock is a constant, since the dividends are based on common stock investment and not patronage. Membership is assumed fixed in the short-run which fixes the amount of common stock investment and, therefore, dividends on common stock. Although, nonmembers can invest in cooperatives and legally receive patronage dividends, I will assume this not to be the case in the model developed here. Any investment in stock by nonmembers can be treated similar to debt, since it normally has a fixed rate of return not dependent upon net savings.

#### Patron Models

The objective of a typical member patron and nonmember patron is normally assumed to be maximization of his profits. As described in the model of the cooperative association, we will assume the patrons market the products they produce in set  $X$ , some within the cooperative association,  $X_c$ , and some outside the cooperative association,  $X_o$ . Similarly,  $Y$  is the set of variable inputs purchased by the patrons, some from the cooperative association,  $Y_c$ , and some from outside the cooperative association,  $Y_o$ .

The typical member patron will want to maximize his total profits, which can be expressed as;

$$\pi_M = \sum_{i \in X} p_i q_i - \sum_{i \in Y} p_i q_i + ds + pvpd - Fc \quad (3.1)$$

where  $p_i$  and  $q_i$  are respectively, the price paid or received and the total quantity of the  $i$ -th product or factor, where  $ds$  is the dividends on the cooperative stock held by the member patron,  $pvpd$  is the present value of the patronage dividends which the member patron expects to receive, and  $Fc$  is the fixed cost of the member firm. The present value of the patronage dividends ( $pvpd$ ) may be restated as;

$$pvpd = [s + \frac{(1-s)}{(1+d)^\tau}] \sum_{i \in c} r_i^* q_i \quad (3.2)$$

where  $s$  is the proportion of patronage dividends paid in cash and  $(1-s)$  is the proportion deferred to a revolving fund of length  $\tau$ . Here,  $c$  represents the products sold to (set  $X$ ) or factors purchased from (set  $Y$ ) the cooperative by the member,  $r_i^*$  represents the expected per unit patronage refund on the  $i$ -th product or factor, and  $d$  represents the discount rate, i.e., it may be the opportunity cost or interest rate paid by the member patron for long term debt.  $r_i^*$  would vary for each member patron and could be dependent on past dividend rates. A function is not specified for  $r_i^*$  here.

Given this objective function, a continuous production function with continuous first-order and second-order derivatives, and the quantity of each of the fixed factors, we can solve for the optimal

production of goods supplied in set X and the optimal amount of inputs demanded in set Y. This can be done by specifying the Lagrangian of the member patron;

$$L_M = \sum_{i \in X} p_i q_i - \sum_{i \in Y} p_i q_i + ds + pvpd - Fc + \lambda_1 [\phi(q_X, q_Y, q_F, q_G)] \\ + \sum_{i \in F} \lambda_{2i} [q_{i0} - q_i] \quad (3.3)$$

where  $\phi(q_X, q_Y, q_F, q_G)$  is the implicit production function with continuous first-order and second-order derivatives,  $q_X$  is the vector of quantities of unprocessed products produced by the member,  $q_Y$  is the vector of quantities of variable inputs used in production by the member,  $q_G$  is the vector of quantities of public goods used in production by the member,  $q_F$  is the vector of quantities of fixed factors used in production by the member, and  $q_{i0}$  ( $i \in F$ ) is the total amount of fixed factors available for production. If the marginal value products of the fixed factors (set F) do not reach zero, the fixed factor will be exhausted in the production process. This assumption does not appear totally unrealistic and will be made here. Imposing this assumption allows the economic implications and results of the model to be more easily derived and understood. The variable  $\lambda_1$  is the Lagrange multiplier associated with the production function constraint and  $\lambda_{2i}$  is the Lagrange multiplier associated with the  $i$ -th fixed factor ( $i \in F$ ).

Given these assumptions, the first-order conditions for the model in 3.3 become;

for all  $i \in X_C$

$$\frac{\partial L_M}{\partial q_i} = p_i + r_i \left[ s + \frac{(1-s)}{(1+d)^T} \right] + \lambda_1 \frac{\partial \phi}{\partial q_i} = 0 \quad (3.4)$$

for all  $i \in X_O$

$$\frac{\partial L_M}{\partial q_i} = p_i + \lambda_1 \frac{\partial \phi}{\partial q_i} = 0 \quad (3.5)$$

for all  $i \in Y_C$

$$\frac{\partial L_M}{\partial q_i} = -p_i + r_i \left[ s + \frac{(1-s)}{(1+d)^T} \right] + \lambda_1 \frac{\partial \phi}{\partial q_i} = 0 \quad (3.6)$$

for all  $i \in Y_O$

$$\frac{\partial L_M}{\partial q_i} = -p_i + \lambda_1 \frac{\partial \phi}{\partial q_i} = 0 \quad (3.7)$$

for all  $i \in F$

$$\frac{\partial L_M}{\partial q_i} = \lambda_1 \frac{\partial \phi}{\partial q_i} - \lambda_{2i} = 0 \quad (3.8)$$

for  $\lambda_1$

$$\frac{\partial L_M}{\partial \lambda_1} = \phi(q_X, q_Y, q_F, q_G) = 0 \quad (3.9)$$

for all  $i \in F$

$$\frac{\partial L_M}{\partial \lambda_{2i}} = q_{i0} - q_i = 0. \quad (3.10)$$

Given these first-order conditions, the member patron will produce the unprocessed product (set X) and sell to the cooperative association at the point where the effective price (price received plus the per unit discounted present value of patronage dividends) received is equal to the marginal cost of producing it,<sup>1</sup> i.e.;

for all  $i \in X_c$

$$p_i + [s + \frac{(1-s)}{(1+d)\tau}]r_i^* = -\lambda_1 \frac{\partial \phi}{\partial q_i}. \quad (3.11)$$

The member patron will produce the unprocessed product (set X) and sell outside the cooperative association at the point where the price received is equal to the marginal cost of producing it,<sup>2</sup> i.e.;

for all  $i \in X_o$

$$p_i = -\lambda_1 \frac{\partial \phi}{\partial q_i}.$$

A similar conclusion may be reached about the demand for inputs (set Y).

The member patron will demand inputs from the cooperative to the point

<sup>1</sup>J. S. Royer (42, pp. 37-40).

<sup>2</sup>J. S. Royer (42, p. 40).

where the effective price is equal to the marginal value product of using the input in production, i.e.;

for all  $i \in Y_C$

$$p_i = [s + \frac{(1-s)}{(1+d)}]r_i^* = -\lambda_1 \frac{\partial \phi}{\partial q_i} . \quad (3.13)$$

The member will purchase inputs outside the cooperative to the point where the price of the input is equal to the marginal value product of using the input in production,<sup>1</sup> i.e.;

for all  $i \in Y_O$

$$p_i = -\lambda_1 \frac{\partial \phi}{\partial q_i} . \quad (3.14)$$

Given the first-order conditions we can solve for the optimal values of the choice variables as functions of the parameters if the bordered Hessian matrix of the model is nonvanishing and negative definite. This results in output supply functions for products in set  $X_C$  and set  $X_O$  and input demand functions for the inputs in set  $Y_C$  and set  $Y_O$ . The general functions can be represented as such;

for  $i \in X_C, Y_C, X_O, Y_O$

$$q_i^M = q_i(p_X, p_Y, R_C^*, Q_G) \quad (3.15)$$

where  $p_X$  is the vector of prices the member receives for his unprocessed products from the cooperative,  $p_Y$  is the vector of prices the member

<sup>1</sup>J. S. Royer (42, pp. 41-42).



pays for the inputs he purchases from the cooperative,  $R_C^*$  is the vector of expected patronage dividend rates on the unprocessed products and inputs the member expects to receive for trading with the cooperative, and  $Q_G$  is the vector of quantities of public goods produced by the cooperative. Similar functions could be developed for those products purchased or sold outside the cooperative association by the member. The total quantity demanded of each of the inputs and the total quantity supplied of each of the unprocessed products by all members will be the horizontal summation across all members for each product, i.e.;

for all  $i \in X, Y$

$$q_i^{TM} = q_i^T(P_X, P_Y, P_C, Q_G) \quad (3.16)$$

where  $q_i^{TM}$  is the total quantity supplied of the unprocessed products (set  $X$ ) by the members or total quantity demanded of the inputs (set  $Y$ ) by the members,  $R_C$  is the vector of expected patronage dividend rates of the members,  $P_X$ ,  $P_Y$ , and  $Q_G$  are as defined before.

A similar model may be developed for the nonmember patron, except that the nonmember patron will not participate in the patronage dividends or dividends on stock, i.e., the profit function of the typical nonmember patron is;

$$\pi_{NM} = \sum_{i \in X} p_i q_i - \sum_{i \in Y} p_i q_i - F_c. \quad (3.17)$$

The nonmember will face the same constraints as the member, i.e., his

Lagrangian becomes;

$$\begin{aligned}
 L_{NM} = & \sum_{i \in X} p_i q_i - \sum_{i \in Y} p_i q_i - F_C + \lambda_1 [\phi(q_X, q_Y, q_F, q_G)] \\
 & + \sum_{i \in F} \lambda_{2i} [q_{io} - q_i].
 \end{aligned} \tag{3.18}$$

From this Lagrangian, we obtain the following first-order conditions;

for all  $i \in X_C$ ,  $X_O$

$$\frac{\partial L_{NM}}{\partial q_i} = p_i + \lambda_1 \frac{\partial \phi}{\partial q_i} = 0, \tag{3.19}$$

for all  $i \in Y_C$ ,  $Y_O$

$$\frac{\partial L_{NM}}{\partial q_i} = -p_i + \lambda_1 \frac{\partial \phi}{\partial q_i} = 0, \tag{3.20}$$

for all  $i \in F$

$$\frac{\partial L_{NM}}{\partial q_i} = \lambda_1 \frac{\partial \phi}{\partial q_i} - \lambda_{2i} = 0, \tag{3.21}$$

for  $\lambda_1$

$$\frac{\partial L_{NM}}{\partial \lambda_1} = \phi(q_X, q_Y, q_F, q_G) = 0, \tag{3.22}$$

for all  $i \in F$

$$\frac{\partial L_{NM}}{\partial \lambda_{2i}} = q_{io} - q_i = 0. \tag{3.23}$$

Similar economic implications may be drawn about the nonmember patrons as were drawn about the member patrons. We may also derive output supply functions for products in set  $X_C$  and set  $X_O$  and input demand functions for the inputs in set  $Y_C$  and set  $Y_O$ . Again, these may be derived from the first-order conditions if the bordered Hessian matrix of the model is nonvanishing and negative definite.

Assuming the bordered Hessian to be nonvanishing and negative definite, we will find the input demand and output supply functions of the typical nonmember patron to be a function of the parameters in the model, i.e.;

for all  $i \in X_C, Y_C$

$$q_i^N = q_i(P_X, P_Y, Q_G) \quad (3.24)$$

where all variables are as defined before. The total quantity supplied by all nonmembers to the cooperative association of the products in set  $X_C$  and the total quantity demanded by all nonmembers from the cooperative association of all inputs in set  $Y_C$  will be the horizontal summation across all nonmembers for each product, i.e.;

for all  $i \in X_C, Y_C$

$$q_i^{TN} = q_i^T(P_X, P_Y, Q_G) \quad (3.25)$$

where  $q_i^{TN}$  is the total quantity supplied (or demanded) of the product from all nonmembers.

### The Cooperative Model

Assuming the relationships in the cooperative association to be as outlined in Figure 2.1, and assuming the objective of the cooperative decision-maker to be that of maximizing the total collective profits of the member patrons, we can specify our objective function as;

$$\Pi = \sum_{i \in X} p_i q_i^{TM} - \sum_{i \in Y} p_i q_i^{TM} - FCM + DS + PVPD \quad (3.26)$$

where  $\Pi$  is the total collective profits of the member patrons,  $q_i^{TM}$  is the total amount of product  $i$  purchased or sold by the member patrons (determined in the patron model),  $p_i$  is the price of product  $i$ , FCM is the total collective fixed costs of the member patrons, DS is the total dividend on members stock, and PVPD is the total present value of patronage dividends.

The total dividend on members stock, DS, can be assumed to be a fixed parameter in the short-run, since this would be a method of distributing the net savings of the cooperative and would affect the financial structure decision facing the cooperative.

PVPD may be redefined as;

$$PVPD = \left[ s + \frac{(1-s)}{(1+d)^T} \right] \left[ \sum_{i \in X, Y} r_i q_{ic}^{TM} \right] \quad (3.27)$$

where  $q_{ic}^{TM}$  is the total amount of product  $i$  traded with the cooperative by the member patrons. Since cooperative associations have the principle of operation at cost, I will assume the net savings less the dividend on members stock is paid out as patronage dividends, i.e.;

$$NS - DS = \sum_{i \in X, Y} r_i q_i^{TM} \quad (3.28)$$

and define NS as;

$$NS = \sum_{i \in Z} p_i q_i + \sum_{i \in Y} p_i q_i^T - \sum_{i \in X} p_i q_i^T - \sum_{i \in V} p_i \sum_{j \in Y, Z, G} q_{ij} \quad (3.29)$$

where  $q_i^T$  represents the total amount of product  $i$  supplied or demanded from the cooperative by both member and nonmember patrons,  $q_i$  ( $i \in Z$ ) is the set of finished products sold outside the cooperative association, and  $q_{ij}$  is the total amount of input  $i$  used to produce product  $j$ . Given these definitions, we can redefine our objective function to be;

$$\Pi = \sum_{i \in X} p_i q_i^{TM} - \sum_{i \in Y} p_i q_i^{TM} + [s + \frac{(1-s)}{(1+d)}] [NS - DS] + DS - FCM \quad (3.30)$$

where all variables are as defined before (including NS in Equation 3.29).

The objective function of the cooperative association (3.30) will be subject to several constraints. A production function must be specified which will have continuous first-order and second-order derivatives. We can show the production function in implicit form as;

$$\phi(Q_Z, Q_Y, Q_G, Q_X, Q_V, Q_{FC}) = 0 \quad (3.31)$$

where  $Q_Z$  is the vector of quantities of processed products in set  $Z$ ,  $Q_Y$  is the vector of quantities of variable factors in set  $Y$ ,  $Q_G$  is the vector of quantities of public goods in set  $G$ ,  $Q_X$  is the vector of quantities of unprocessed products in set  $X$  used to produce products in set  $Y$ , set  $Z$ , and set  $G$ ,  $Q_V$  is the vector of quantities of variable

inputs in set V used to produce products in set Y, set Z, and set G, and  $Q_{FC}$  is the vector of quantities of fixed factors of production used to produce products in set Y, set Z, and set G.

A second constraint which is specified maintains that all of the unprocessed product (sex X) purchased from the member and nonmember patrons is transformed into final product, i.e.;

for all  $i \in X$

$$q_i^T - \sum_{j \in Y, Z, G} q_{ij} = 0. \quad (3.32)$$

We may also make the same assumption concerning the fixed factors of production which we made for the patron models, that the fixed factors of production are exhausted in the production process, i.e., we get the following constraint;

for all  $i \in FC$

$$q_{i0} - \sum_{j \in Y, Z, G} q_{ij} = 0 \quad (3.33)$$

where  $q_{i0}$  ( $i \in FC$ ) is the total amount of fixed factor i available for production.

The final constraint for the analysis will be that a specified amount of capital from the net savings is deferred for a period of  $\tau$  years. This constraint is derived from the long-run financial structure decision made by the cooperative association, i.e.,

$$\bar{K} = (1-s)(NS-DS) \quad (3.34)$$

where  $\bar{K}$  is the total amount of net savings to be deferred (determined in

the long-run financial decision model) and all other variables are as defined before. This constraint and Equation 3.29 assure a consistency between the cooperative short-run and long-run decisions, i.e., the level of net savings which is determined by the solution of the variables in the long-run decision model is a constraint which must be met in the short-run decision model. The constraint in Equation 3.34 can be rewritten as;

$$(1-s)NS = \bar{K} + (1-s)DS. \quad (3.35)$$

Equation 3.35 is a constraint for the short-run model developed in this chapter which Royer did not use in the production and pricing decision model he developed. This constraint is the only major difference in the two models.

#### The Lagrangian Function and First-order Conditions

From the previously defined objective function, (3.30), and the constraints (3.31, 3.32, 3.33, and 3.35), we can define the Lagrangian for this submodel to be;

$$\begin{aligned} L = & \sum_{i \in X} p_i q_i^{TM} - \sum_{i \in Y} p_i q_i^{TM} + [s + \frac{(1-s)}{(1+d)}\tau] [NS-DS] + DS - FCM \\ & + \psi_1 [\bar{\Phi}(Q_Z, Q_Y, Q_G, Q_X, Q_V, Q_{FC})] + \sum_{i \in X} \psi_{2i} [q_i^T - \sum_{j \in Y, Z, G} q_{ij}] \\ & + \sum_{i \in FC} \psi_{3i} [q_{io} - \sum_{j \in Y, Z, G} q_{ij}] \\ & + \psi_4 [\bar{K} + (1-s)DS - (1-s)NS] \end{aligned} \quad (3.36)$$

where NS is defined by Equation 3.29; the  $q_i^{TM}$  ( $i \in X, Y$ ) are as defined in the member patron model (Equation 3.16);  $\Psi_1, \Psi_{2i}$  ( $i \in X$ ),  $\Psi_{3i}$  ( $i \in FC$ ), and  $\Psi_4$  are the Lagrange multipliers corresponding to each of the constraints; and all other variables are as defined before. The instrument variables available to the cooperative association decision-maker are the prices it sets for products in set X and set Y, the quantities of products produced in set Z and set G, and the quantities of inputs in set X, set V, and set FC used in the production of each product in set Y, set Z, and set G. The first-order conditions are;

for all  $j \in X$

$$\begin{aligned} \frac{\partial L}{\partial p_j} = & q_j^{TM} + \sum_{i \in X} p_i \frac{\partial q_i^{TM}}{\partial p_j} - \sum_{i \in Y} p_i \frac{\partial q_i^{TM}}{\partial p_j} + [s + \frac{(1-s)}{(1+d)}] [\sum_{i \in Y} p_i \frac{\partial q_i^T}{\partial p_j} \\ & - q_j^T - \sum_{i \in X} p_i \frac{\partial q_i^T}{\partial p_j}] + \sum_{i \in X, Y} \Psi_1 \frac{\partial \phi}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j} + \sum_{i \in X} \Psi_{2i} \frac{\partial q_i^T}{\partial p_j} \\ & + \Psi_4 [-(1-s) (\sum_{i \in Y} p_i \frac{\partial q_i^T}{\partial p_j} - q_j^T - \sum_{i \in X} p_i \frac{\partial q_i^T}{\partial p_j})] = 0, \end{aligned} \quad (3.37)$$

for all  $j \in Y$

$$\begin{aligned} \frac{\partial L}{\partial p_j} = & \sum_{i \in X} p_i \frac{\partial q_i^{TM}}{\partial p_j} - q_j^{TM} - \sum_{i \in Y} p_i \frac{\partial q_i^{TM}}{\partial p_j} + [s + \frac{(1-s)}{(1+d)}] [q_j^T + \sum_{i \in Y} p_i \frac{\partial q_i^T}{\partial p_j} \\ & - \sum_{i \in X} p_i \frac{\partial q_i^T}{\partial p_j}] + \sum_{i \in X, Y} \Psi_1 \frac{\partial \phi}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j} + \sum_{i \in X} \Psi_{2i} \frac{\partial q_i^T}{\partial p_j} \\ & + \Psi_4 [-(1-s) (q_j^T + \sum_{i \in Y} p_i \frac{\partial q_i^T}{\partial p_j} - \sum_{i \in X} p_i \frac{\partial q_i^T}{\partial p_j})] = 0, \end{aligned} \quad (3.38)$$



for all  $j \in Z$

$$\frac{\partial L}{\partial q_j} = [s + \frac{(1-s)}{(1+d)^T}] [p_j + q_j \frac{\partial p_j}{\partial q_j}] + \psi_1 \frac{\partial \phi}{\partial q_j} + \psi_4 [-(1-s) (p_j + q_j \frac{\partial p_j}{\partial q_j})] = 0, \quad (3.39)$$

for all  $j \in G$

$$\begin{aligned} \frac{\partial L}{\partial q_j} = & \sum_{i \in X} p_i \frac{\partial q_i^{TM}}{\partial q_j} - \sum_{i \in Y} p_i \frac{\partial q_i^{TM}}{\partial q_j} + [s + \frac{(1-s)}{(1+d)^T}] \\ & [ \sum_{i \in Y} p_i \frac{\partial q_i^T}{\partial q_j} - \sum_{i \in X} p_i \frac{\partial q_i^T}{\partial q_j} ] + \sum_{i \in X, Y} \psi_1 \frac{\partial \phi}{\partial q_i^T} \frac{\partial q_i^T}{\partial q_j} + \psi_1 \frac{\partial \phi}{\partial q_j} \\ & + \sum_{i \in Z} \psi_{2i} \frac{\partial q_i^T}{\partial q_j} + \psi_4 [-(1-s) ( \sum_{i \in Y} p_i \frac{\partial q_i^T}{\partial q_j} - \sum_{i \in X} p_i \frac{\partial q_i^T}{\partial q_j} )] = 0, \end{aligned} \quad (3.40)$$

for all  $i \in X \quad j \in Y, Z, G$

$$\frac{\partial L}{\partial q_{ij}} = \psi_1 \frac{\partial \phi}{\partial q_{ij}} - \psi_{2i} = 0, \quad (3.41)$$

for all  $i \in V \quad j \in Y, Z, G$

$$\begin{aligned} \frac{\partial L}{\partial q_{ij}} = & [s + \frac{(1-s)}{(1+d)^T}] [-p_i - q_{ij} \frac{\partial p_i}{\partial q_{ij}}] + \psi_1 \frac{\partial \phi}{\partial q_{ij}} + \\ & + \psi_4 [-(1-s) (-p_i - q_{ij} \frac{\partial p_i}{\partial q_{ij}})] = 0, \end{aligned} \quad (3.42)$$

for all  $i \in FC \quad j \in Y, Z, G$

$$\frac{\partial L}{\partial q_{ij}} = \psi_1 \frac{\partial \phi}{\partial q_{ij}} - \psi_{3i} = 0, \quad (3.43)$$

for  $\Psi_1$

$$\frac{\partial L}{\partial \Psi_1} = \phi(Q_Z, Q_Y, Q_G, Q_X, Q_V, Q_{FC}) = 0, \quad (3.44)$$

for all  $j \in X$

$$\frac{\partial L}{\partial \Psi_{2j}} = q_j^T - \sum_{i \in Y, Z, G} q_{ij} = 0, \quad (3.45)$$

for all  $j \in FC$

$$\frac{\partial L}{\partial \Psi_{3j}} = q_{j0} - \sum_{i \in Y, Z, G} q_{ji} = 0, \quad (3.46)$$

for  $\Psi_4$

$$\frac{\partial L}{\partial \Psi_4} = \bar{K} + (1-s)DS - (1-s)NS = 0. \quad (3.47)$$

An interpretation of results comparable to these was given by Royer<sup>1</sup> in his analysis of his general nonlinear programming model of a producers cooperative. Equation 3.37 may be written as;

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<sup>1</sup>J. S. Royer (42, pp. 85-104).

for all  $j \in X$

$$\begin{aligned}
 \frac{\partial L}{\partial p_j} = & (p_j + q_j^{TM} \frac{\partial p_j}{\partial q_j^{TM}}) \frac{\partial q_j^{TM}}{\partial p_j} + \sum_{\substack{i \in X \\ i \neq j}} p_i \frac{\partial q_i^{TM}}{\partial p_j} - \sum_{i \in Y} p_i \frac{\partial q_i^{TM}}{\partial p_j} \\
 & - [s + \frac{(1-s)}{(1+d)} \tau] [ (p_j + q_j^T) \frac{\partial p_j}{\partial q_j^T} \frac{\partial q_j^T}{\partial p_j} + \sum_{\substack{i \in X \\ i \neq j}} p_i \frac{\partial q_i^T}{\partial p_j} - \sum_{i \in Y} p_i \frac{\partial q_i^T}{\partial p_j} ] \\
 & + \sum_{i \in X, Y} \psi_1 \frac{\partial \phi}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j} \\
 & + \sum_{i \in X} \psi_{2i} \frac{\partial q_i^T}{\partial p_j} \\
 & + \psi_4 \{ (1-s) [ (p_j + q_j^T) \frac{\partial p_j}{\partial q_j^T} \frac{\partial q_j^T}{\partial p_j} + \sum_{\substack{i \in X \\ i \neq j}} p_i \frac{\partial q_i^T}{\partial p_j} \\
 & - \sum_{i \in Y} p_i \frac{\partial q_i^T}{\partial p_j} ] \} = 0. \tag{3.48}
 \end{aligned}$$

The term  $p_j + q_j^{TM} (\partial p_j / \partial q_j^{TM})$  may be interpreted as the variation in total private revenues from the  $j$ -th product arising from output shifts which are induced by a variation in the  $j$ -th price ( $dp_j$ ).

Royer represents this effect as  $(\partial TPR / \partial q_j^{TM}) (\partial q_j^{TM} / \partial p_j)$ . The term

$\sum_{\substack{i \in X \\ i \neq j}} p_i (\partial q_i^{TM} / \partial p_j)$  may be interpreted as the variation in total private revenues from all other products in set  $X$  arising from output shifts

induced by  $dp_j$  and is represented by Royer as  $\sum_{\substack{i \in X \\ i \neq j}} (\partial TPR / \partial q_i^{TM}) (\partial q_i^{TM} / \partial p_j)$ .

Similarly, the term  $\sum_{i \in Y} p_i (\partial q_i^{TM} / \partial p_j)$  can be interpreted as the variation in total private costs arising from shifts in factor use induced by  $dp_j$

and is represented by Royer as  $\sum_{i \in Y} (\partial \text{TPC} / \partial q_i^{\text{TM}}) (\partial q_i^{\text{TM}} / \partial p_j)$ .

The terms in the second line of Equation 3.48 may be interpreted with similar economic notation. The difference in these terms lie in: 1) the addition of  $s + (1-s)/(1+d)^T$ ; 2) the substitution of  $q_i^T$  for each  $q_i^{\text{TM}}$ , and; 3) the fact that what was a revenue in the first line of the equation is now a cost and what was a cost is now a revenue. The cost and revenue terms are turned around (costs for revenues, revenues for costs) because of the affect of the products on net savings in the cooperative. The products in set X represent a cost to the cooperation in determining the net savings and the products in set Y represent a revenue. The term  $q_i^T$  is substituted for  $q_i^{\text{TM}}$  since the total collective product traded by member and nonmembers determines the profits of the cooperative. Finally, the  $s+(1-s)/(1+d)^T$  term is added to represent the present value affect of net savings to the members. The net savings are allocated to the members and represent an addition to the total collective profits of the members.

Given these differences, the term  $[s+(1-s)/(1+d)^T] [p_j + q_j^T (\partial p_j / \partial q_j^T)] (\partial q_j^T / \partial p_j)$  is interpreted as the variation in total collective costs from the j-th product arising from changes in the quantity supplied which is induced by  $dp_j$ . Royer represents this as  $(\partial \text{TCC} / \partial q_j^T) (\partial q_j^T / \partial p_j)$ . The term for all other products in set X may be interpreted similarly and is represented by Royer as  $\sum_{\substack{i \in X \\ i \neq j}} (\partial \text{TCC} / \partial q_i^T) (\partial q_i^T / \partial p_j)$ . The term for products in set Y represents the variation in total collective revenues arising from changes in quantities demanded which are induced by  $dp_j$ .

Royer represents these as  $\sum_{i \in Y} (\partial TCR / \partial q_i^T) (\partial q_i^T / \partial p_j)$ .

Royer interprets the term which is located in the third line of Equation 3.48 as the variation in the profits of the members from the production of the products in set X and set Y arising from changes in the quantities demanded or supplied which are induced by  $\partial p_j$ . He represents the term as  $\sum_{i \in X} (\partial TMP / \partial q_i^T) (\partial q_i^T / \partial p_j) - \sum_{i \in Y} (\partial TMP / \partial q_i^T) (\partial q_i^T / \partial p_j)$ .

The term in the fourth line of Equation 3.48 represents the variation in the profits of the members from the transformation of products in set X to final product arising from a change in the quantities supplied which are induced by  $\partial p_j$  and may be represented by  $-\sum_{i \in X} (\partial TMP / \partial q_i^T) (\partial q_i^T / \partial p_j)$ . This term is not found in Royer's model because he doesn't include the constraints corresponding to it. This term is analogous to the term derived from the third line of the equation. Inclusion of this constraint would permit the possibility of the firm utilizing a product in set X in the production process when there may be negative value to the profits of the member patrons.

The final terms, located in the fifth line of Equation 3.48, represent the variation in the profits of the members from the change in capital employed via deferred patronage dividends arising from output shifts which are induced by  $\partial p_j$ . The affect of the output shifts on capital employed is through the net savings of the cooperative. The terms inside the { } of the fifth line represent the change in the amount of deferred patronage dividends induced by  $\partial p_j$  and may be represented by  $\sum_{i \in X, Y} (\partial DP / \partial q_i^T) (\partial q_i^T / \partial p_j)$ .

The Lagrange multiplier  $\Psi_4$  may be interpreted as the variation in the total collective profits of the members induced by a change in the amount of deferred patronage required, and may be represented by  $(\partial \text{TMP} / \partial \text{DP})$ . Specifying the terms in the  $\{ \}$  and the  $\Psi_4$  in this manner allows the term in the fifth line to be interpreted as  $(\partial \text{TMP} / \partial \text{DP}) [\sum (\text{DP} / \partial q_i^T) (\partial q_i^T / \partial p_j)]$ .

Specifying the terms in Equation 3.48 in this manner allows us to rewrite the equation as;

for all  $j \in X$

$$\begin{aligned}
 \frac{\partial L}{\partial p_j} = & \left( \sum_{i \in X} \frac{\partial \text{TPR}}{\partial q_i^{\text{TM}}} \frac{\partial q_i^{\text{TM}}}{\partial p_j} - \sum_{i \in Y} \frac{\partial \text{TPC}}{\partial q_i^{\text{TM}}} \frac{\partial q_i^{\text{TM}}}{\partial p_j} \right) \\
 & + \left( \sum_{i \in Y} \frac{\partial \text{TCR}}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j} - \sum_{i \in X} \frac{\partial \text{TCC}}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j} \right) \\
 & \left( \sum_{i \in X} \frac{\partial \text{TMP}}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j} - \sum_{i \in Y} \frac{\partial \text{TMP}}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j} \right) \\
 & - \left( \sum_{i \in X} \frac{\partial \text{TMP}}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j} \right) + \left( \sum_{i \in X, Y} \frac{\partial \text{TMP}}{\partial \text{DP}} \frac{\partial \text{DP}}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j} \right) = 0. \quad (3.49)
 \end{aligned}$$

This is equivalent to stating that, for a maximum, the sum of the variation in total private profits arising from input and output shifts induced by  $dp_j$ ; the variation in total collective profits arising from changes in quantities supplied and demanded induced by  $dp_j$ ; the variation in the profits of the members arising from changes in the cooperative's production induced by  $dp_j$ ; the variation in the

profits of the members from the transformation of products in set X to final product arising from a change in the quantities supplied induced by  $dp_j$ ; and the variation in the profits of the members from the change in capital employed via deferred patronage dividends arising from output shifts induced by  $dp_j$  must equal zero.

A condition similar to Equation 3.49 may be derived from Equation 3.38. Equation 3.38 may be rewritten as;

for all  $j \in Y$

$$\begin{aligned}
 \frac{\partial L}{\partial p_j} = & \sum_{i \in X} p_i \frac{\partial q_i^{TM}}{\partial p_j} - (p_j + q_j^{TM}) \frac{\partial p_j}{\partial q_j^{TM}} \frac{\partial q_j^{TM}}{\partial p_j} - \sum_{\substack{i \in Y \\ i \neq j}} p_i \frac{\partial q_i^{TM}}{\partial p_j} \\
 & - [s + \frac{(1-s)}{(1+d)^T}] [ \sum_{i \in X} p_i \frac{\partial q_i}{\partial p_j} - (p_j + q_j^T) \frac{\partial p_j}{\partial q_j^T} \frac{\partial q_j^T}{\partial p_j} - \sum_{\substack{i \in Y \\ i \neq j}} p_i \frac{\partial q_i^T}{\partial p_j} \\
 & + \sum_{i \in X, Y} \psi_1 \frac{\partial \phi}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j} \\
 & + \sum_{i \in X} \psi_{2i} \frac{\partial q_i^T}{\partial p_j} \\
 & + \psi_4 \{ (1-s) [ \sum_{i \in X} p_i \frac{\partial q_i^T}{\partial p_j} - (p_j + q_j^T) \frac{\partial p_j}{\partial q_j^T} \frac{\partial q_j^T}{\partial p_j} - \sum_{\substack{i \in Y \\ i \neq j}} p_i \frac{\partial q_i^T}{\partial p_j} ] \} = 0. \quad (3.50)
 \end{aligned}$$

Using the interpretations of the terms derived for Equation 3.49,

Equation 3.50 may be rewritten as;

for all  $j \in Y$

$$\begin{aligned}
 \frac{\partial L}{\partial p_j} = & \left( \sum_{i \in X} \frac{\partial TPR}{\partial q_i^{TM}} \frac{\partial q_i^{TM}}{p_j} - \sum_{i \in Y} \frac{\partial TPC}{\partial q_i^{TM}} \frac{\partial q_i^{TM}}{\partial p_j} \right. \\
 & + \left[ \sum_{i \in Y} \frac{\partial TCR}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j} - \sum_{i \in X} \frac{\partial TCC}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j} \right) \\
 & + \left( \sum_{i \in X} \frac{\partial TMP}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j} - \sum_{i \in Y} \frac{\partial TMP}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j} \right) \\
 & - \left( \sum_{i \in X} \frac{\partial TMP}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j} \right) \\
 & + \left( \sum_{i \in X, Y} \frac{\partial TMP}{\partial DP} \frac{\partial DP}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j} \right) = 0.
 \end{aligned} \tag{3.51}$$

Equation 3.51 yields an interpretation similar to that for Equation 3.50.

In Equation 3.39, the term  $p_j + q_j \frac{\partial p_j}{\partial q_j}$  is the marginal revenue of the  $j$ -th product in set  $Z$ . The term  $\Psi_1 \frac{\partial \Phi}{\partial q_j}$  was interpreted by Royer as the variation in the profits of the members arising from a change in the quantity of the output produced by the cooperative ( $dq_j$ ). The term  $\Psi_4 [-(1-s)(p_j + q_j \frac{\partial p_j}{\partial q_j})]$  can be interpreted as the variation in the profits of the members arising from a change in the amount of deferred patronage dividends which is induced by a change in the quantity of output produced. Thus, the interpretation for Equation



3.39 is equivalent to stating that, for a maximum, the marginal revenue to the cooperative, multiplied by  $s + (1-s)/(1+d)^T$ , must be equal to the variation in the profits of the members arising from a change in quantity of output produced and a change in the amount of deferred patronage dividends which is induced by  $dq_j$ .

Equation 3.40 may be interpreted in the same manner as were Equations 3.37 and 3.38, except that shifts are caused by  $dq_j (j \in G)$ . Equation 3.40 may be rewritten as;

for all  $j \in G$

$$\begin{aligned}
 \frac{\partial L}{\partial q_j} = & \left( \sum_{i \in X} \frac{\partial TPR}{\partial q_i^{TM}} \frac{\partial q_i^{TM}}{\partial q_j} - \sum_{i \in Y} \frac{\partial TPC}{\partial q_i^{TM}} \frac{\partial q_i^{TM}}{\partial q_j} \right) \\
 & + \left( \sum_{i \in Y} \frac{\partial TCR}{\partial q_i^T} \frac{\partial q_i^T}{\partial q_j} - \sum_{i \in X} \frac{\partial TCC}{\partial q_i^T} \frac{\partial q_i^T}{\partial q_j} \right) \\
 & + \left( \sum_{i \in X} \frac{\partial TMP}{\partial q_i^T} \frac{\partial q_i^T}{\partial q_j} - \sum_{i \in Y} \frac{\partial TMP}{\partial q_i^T} \frac{\partial q_i^T}{\partial q_j} \right) \\
 & - \left( \sum_{i \in X} \frac{\partial TMP}{\partial q_i^T} \frac{\partial q_i^T}{\partial q_j} \right) \\
 & + \left( \sum_{i \in X, Y} \frac{\partial TMP}{\partial DP} \frac{\partial DP}{\partial q_i^T} \frac{\partial q_i^T}{\partial q_j} \right) = 0. \tag{3.52}
 \end{aligned}$$

This yields an interpretation similar to those given in Equations 3.49 and 3.51.

For Equation 3.41 the term  $\Psi_1 (\partial \Phi / \partial q_{ij})$  is the variation in the profits of the members from a change in the quantity of the  $i$ -th factor

of set  $X$  used to produce output  $j$  in the cooperative ( $dq_{ij}$ ).  $\Psi_{2i}$  may be interpreted as the variation in the profits of the member patrons from a change in the amount of unprocessed product  $j$  used in the cooperative. Thus, Equation 3.41 implies that the variation in the profits of the members from  $dq_{ij}$  must be equal for all  $j$ .

In Equation 3.42, Royer interprets the term  $-p_i - q_{ij} (\partial p_i / \partial q_{ij})$  as the marginal factor cost to the cooperative of using the  $i$ -th variable input. Again, the term  $\Psi_1 (\partial \Phi / \partial q_{ij})$  is interpreted as the variation in the profits of the members from a change in the quantity of input  $i$  used to produce output  $j$  by the cooperative ( $dq_{ij}$ ). The term  $\Psi_4 - (1-s)[-p_i - q_{ij} (\partial p_i / \partial q_{ij})]$  can be interpreted as the variation in the profits of the members from a change in the amount of required deferred patronage dividends which is induced by  $dq_{ij}$ . Thus, Equation 3.42 may be interpreted as stating that, for a maximum, the variation in the profits of the members from  $dq_{ij}$  must be equal to the marginal factor cost of the input multiplied by  $s + (1-s)/(1+d)^T$ . This condition implies that the variation in the profits of the members from  $dq_{ij}$  must be equal for all  $j$ .

In the analysis of the condition stated in Equation 3.43, Royer again interprets  $\Psi_1 (\partial \Phi / \partial q_{ij})$  as the variation in the profits of the members from a change in the quantity of input  $i$  used to produce output  $j$  by the cooperative ( $dq_{ij}$ ).  $\Psi_{3i}$  was interpreted by Royer as the imputed value or shadow price of the  $i$ -th fixed factor. Thus, Equation 3.41 is interpreted as equivalent to stating that, for a maximum, the

variation in member profits from  $dq_{ij}$  must equal the imputed value to the cooperative of the factor. Again, this result implies that the variation in the profits of the members from  $dq_{ij}$  must be equal for all  $j$ .

The first-order conditions stated in Equations 3.44 through 3.47 are just restatements of the constraints of the model. The values of the Lagrange multipliers associated with these constraints indicate the imputed values of the constraints on the cooperative.

The interpretation of the first-order conditions indicate the normative purpose of Royer's study. In his study, he proposed a set of assumptions by which a cooperative association should operate. The model derived from these assumptions allowed Royer to interpret the conditions which maximized the total collective profits of the member patrons. The interpretations derived for the first-order conditions stated in Equations 3.37 through 3.47 are those given by Royer with a few minor changes.

It is not the purpose of this study to analyze the interpretations derived by Royer. This section of the study will focus on the positive purpose of describing the way cooperative associations operate in reality. The model described by Royer was used as a basis for the model derived in this chapter. The model described in this chapter is considered by the author to be a useful tool for the positive purpose of an empirical analysis of how cooperatives actually operate.

The only major difference between the model developed in this

chapter and the one presented by Royer is the addition of the constraint stated in Equation 3.35. This constraint was added to assure a consistency between the short-run and long-run decisions made by the cooperative decision-maker.

CHAPTER IV. COOPERATIVE PRICING AND PRODUCTION SUB-MODEL  
ANALYSIS - EMPIRICAL PROCEDURES

The first-order conditions (Equations 3.37-3.47) represent a set of structural equations which may be solved to yield a set of general reduced form functions for the instrument variables of the model. The instrument variables of most interest in this model include the prices the cooperative pays for the unprocessed products (set X) and the prices it charges for the variable inputs (set Y). The general reduced form functions for these instrument variables may be expressed as;

for all  $j \in X$

$$P_j = P_j(s, \tau, d, R_C, DS, Q_{FC}, P_V, P_Z, \bar{K}), \quad (4.1)$$

for all  $j \in Y$

$$P_j = P_j(s, \tau, d, R_C, DS, Q_{FC}, P_V, P_Z, \bar{K}). \quad (4.2)$$

Differential equations may be derived for the structural equations of the model.<sup>1</sup> The differential equations can often be used to solve for instrument variables and to hypothesize the effect of a change in one of the exogenous variables on an instrument variable. The generality and size of the model presented here makes it impossible to hypothesize signs a priori. The analysis does allow us to recognize variables to use in the empirical procedure. The general reduced form equations for the prices of unprocessed products in set X (4.1) and the

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<sup>1</sup>The differential equations were derived by the author and may be obtained upon request from John Van Sickle, Food and Resource Economics Department, McCarty Hall, University of Florida, Gainesville, Florida, 32611.

variable inputs in set Y (4.2) specify the exogenous variables which are to be tested in the empirical analysis.

#### Data

This phase of the study deals with the empirical estimation of the reduced form functions given in Equations 4.1 and 4.2. The typical cooperative association which is involved in this study purchases grain (set X) from member and nonmember patrons and sells inputs such as feed, fertilizer, and petroleum (set Y) to the member and nonmember patrons.

Information was collected from sixty-eight Iowa cooperatives in the summer of 1979. The sampling procedure for selection of cooperatives was the event of being independently selected from the membership of the Iowa Grain and Feed Dealers Association, conditional on the basis that the firm was a cooperative association.

The sample information for each firm<sup>1</sup> consisted of time series price data for corn and soybeans (set X), a cross-sectional price for feed and fertilizer (set Y), financial data (income statement and balance sheet information), and information about the physical facilities of the cooperative. The time series price data for corn and soybeans included the cooperative association cash bid price for each product on each Thursday from January 6, 1977, through May 31, 1979.

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<sup>1</sup>The questionnaire used in the sample is presented by J. Van Sickle (56).

In order to eliminate the industry supply and demand forces from the analysis, the Chicago Board of Trade closing bid price for the nearby option for each Thursday was subtracted from the cash bid of the cooperative association. This makes the variable for the analysis procedure the cooperative "basis" level on each Thursday.

The cross-sectional price data for feed consisted of the charge for a one ton bulk load, FOB, of; 40% protein hog concentrate meal ration, 35% all natural protein cattle concentrate pellets ration, and 50% protein (containing nonprotein nitrogen) cattle concentrate pellets ration. The cross-sectional price for fertilizer consisted of the charge for anhydrous ammonia in May, 1979. The high, low, and average for each of these cross-sectional variables is given in Table 1.

Table 1. Summary of feed and fertilizer prices

Variable	High	Low	Average
40% hog ration	\$322.00	\$233.40	\$272.78
35% cattle ration	304.00	201.00	255.67
50% cattle ration	278.32	133.31	233.12
Anhydrous ammonia	188.10	150.00	169.90

# Empirical Models for Estimation

Given the time series information across cross-sectional units, we will be concerned with estimating the effects of the exogenous variables in Equation 4.1 on the basis levels. We can denote the bean-basis model for estimation as:

$$BB_{it} = A_0 + \sum_{k=1}^K A_k X_{ikt} + \sum_{\ell=1}^L \gamma_{\ell} Z_{i\ell} + \sum_{k=1}^K \sum_{\ell=1}^L (A\gamma)_{k\ell} X_{ikt} Z_{i\ell} + \epsilon_{it} \quad i = 1, \dots, 68 \quad t = 1, \dots, t_i \quad k = 1, \dots, K \quad (4.3)$$

and the corn-basis model as;

$$BC_{it} = B_0 + \sum_{k=1}^K B_k X_{ikt} + \sum_{\ell=1}^L \rho_{\ell} Z_{i\ell} + \sum_{k=1}^K \sum_{\ell=1}^L (B\rho)_{k\ell} X_{ikt} Z_{i\ell} + \eta_{it} \quad i = 1, \dots, 68 \quad t = 1, \dots, t_i \quad k = 1, \dots, K \quad (4.4)$$

where  $BB_{it}$  is the soybean basis level for the  $i$ -th firm in period  $t$ ,  $A_0$  is the intercept for the soybean basis level,  $X_{ikt}$  is the  $k$ -th time series exogenous variable for the  $i$ -th firm in period  $t$ ,  $A_k$  is the coefficient for  $X_{ikt}$  in the soybean basis model,  $Z_{i\ell}$  is the  $\ell$ -th cross-sectional exogenous variable for the  $i$ -th firm,  $\gamma_{\ell}$  is the coefficient for  $Z_{i\ell}$  in the soybean basis model, and  $(A\gamma)_{k\ell}$  is the interaction coefficient for the terms  $X_{ikt}$  and  $Z_{i\ell}$  in the soybean basis model.  $B_0$ ,  $B_k$ ,  $\rho_{\ell}$ , and  $(B\rho)_{k\ell}$  are the corresponding coefficients in the corn basis model.  $\epsilon_{it}$  is the error term for the  $i$ -th firm in period



t for the soybean basis model and  $\eta_{it}$  is the corresponding error term for the corn basis model. The basic assumptions which can be added are;

Each of the explanatory variables is nonstochastic,  
i.e.,  $X_{ikt}$  and  $Z_{i\ell}$  are known and measured without error. (4.5)

The number of observations exceeds the number of  
coefficients to be estimated. (4.6)

No exact linear relation exists among any of the  
explanatory variables. (4.7)

The behavior of the error terms will not be specified, except to  
say that they have cross-sectional and cross-product independence,  
i.e.;

$$E(\epsilon_{it}) = E(\eta_{it}) = 0 \quad (4.8)$$

$$E(\epsilon_{it}^2) = \sigma_i^2, \quad E(\eta_{it}^2) = \sigma_{\eta i}^2 \quad (4.9)$$

$$E(\epsilon_{it}\epsilon_{jt}) = E(\eta_{it}\eta_{jt}) = E(\epsilon_{it}\eta_{jt}) = 0 \quad i \neq j. \quad (4.10)$$

The data in the feed and fertilizer price analysis are cross-  
sectional, i.e., the model may be stated as;

$$FP_i = c + \sum_{\ell=1}^L C_{\ell} Z_{i\ell} + e_i \quad i = 1, \dots, 68 \quad (4.11)$$

where  $FP_i$  is the 40% hog ration price,  $c$  is the intercept for the  
model,  $Z_{i\ell}$  is the  $\ell$ -th exogenous variable for the  $i$ -th firm,  $C_{\ell}$   
is the coefficient for  $Z_{i\ell}$ , and  $e_i$  is the error term for the  $i$ -th

firm. Similar models may be specified for the 35% cattle ration price, 50% cattle ration price, and anhydrous ammonia fertilizer price. In this model we can assume that:

$$\text{The } Z_{il} \text{ are fixed and measured without error} \quad (4.12)$$

$$E(e_i) = 0 \quad (4.13)$$

$$\begin{aligned} E(e_i e_j) &= \sigma^2 & (i=j) \\ &= 0 & (i \neq j). \end{aligned} \quad (4.14)$$

#### Empirical Procedure

##### Feed and fertilizer price models

Given the data available for the feed and fertilizer price models, ordinary least squares would be an appropriate method of estimation. The procedure followed in the estimation process was to choose those exogenous variables ( $Z_{il}$ ) for the regression analysis which best represented the categories of general exogenous variables in the general reduced form Equation 4.2.

The exogenous variables were chosen from the facility and financial data information collected from each firm, and also from farm production data compiled in the county where the firm's home office was located. These farm production data were taken from the Iowa Agricultural Statistics 1979 compiled and issued by the Iowa Crop and Livestock Reporting Service.

Some of the exogenous variables were represented by the actual

variable as measured in the survey. Other exogenous variables were represented by proxy variables. Because many of the variables measured for the analysis were proxy variables, they represented one or more of the specific variables in Equation 4.2.

The variable  $s$  is the portion of patronage dividends which is paid in cash. Information on this variable was collected from each firm. In most firms, the practice has been to determine a value of  $s$  and use it for several years. In the estimation of the pricing models the firm's most recent value of  $s$  was the value used in the analysis.

The variable  $\tau$  is the number of years the retained portion of the patronage dividend is deferred. The value of this variable was determined by observing the most recent payment of deferred patronage dividends and noting the year in which these dividends had been declared. The number of years away from the current fiscal year in May, 1979, was determined to be the value for  $\tau$ . Some firms paid out the deferred patronage dividend as preferred stock. When this is done, it is common practice to pay cash for the preferred stock on a revolving basis as is done for deferred patronage dividends, but not necessarily by paying the oldest first. When the oldest was paid first, the variable  $\tau$  was calculated in the same manner as for deferred patronage dividends. When the oldest wasn't paid first, a value of zero was given to the variable  $\tau$  and a value of one was given to a dummy variable,  $TD$ . The value of  $TD$  was set to zero in the observations where  $\tau$  was positive. By constructing these variables,  $\tau$

and TD, we were able to measure the affect of  $\tau$  and the general affect of paying for preferred stock in any other manner than the oldest first. In the analysis, we could estimate coefficients for  $\tau$  and TD.

The variable  $d$  is the discount rate given to the deferred portion of patronage dividends by the cooperative. It was assumed that the discount rate was equal for all firms and therefore  $d$  was not used in the analysis.

$R_C$  is the vector of expected per unit patronage dividend rates of the member patrons in the cooperative. Since this vector was impossible to measure, it was assumed that the vector was a function of past rates. It is possible that the cooperative decision-makers can influence  $R_C$  with additional information given to the member patrons, but since this information was impossible to measure, the variable  $R_C$  was constructed only as a function of past patronage refund rates. The expected rate was computed as a simple average of the three most recent actual rates paid out by the  $i$ -th cooperative association for the  $j$  th commodity,  $R_{ij}$ .

The variable DS is the total amount of net savings allocated as dividends on common stock. This variable was not used in the analysis since no cooperative association in the sample had declared a dividend on common stock.

The variable  $Q_{FC}$  is the vector of fixed factors of production available to the cooperative firm. These factors were represented by measures of physical elements owned by the cooperative, e.g., total capacities of production for each of the products, total investment in

fixed assets in both book value and purchase value, number of locations or plants, etc.

The variable  $P_v$  is the vector of prices for the variable inputs purchased outside the cooperative association. These variables would represent the unit prices of the various inputs the cooperative purchases outside its association. Information on these variables was not collected in the sample. The cost of operation was the proxy variable used to indicate these variables. Although the cost of operation also represents other variables, it was chosen as the best representation of variable input prices. The total expenses as measured in the most recent annual income statement were allocated to the products in the analysis. The allocation was based on a factor computed by dividing total dollar sales for the product by total cooperative sales. This factor was then multiplied by total expenses to get a computed cost of operation for the specific product. This cost of operation would be price determined since the allocation of expenses is based on dollar sales.

The variable  $P_z$  is the vector of prices for the final outputs sold outside the cooperative association. These variables would represent the prices that are available for products the cooperative produces and sells outside the cooperative association. Grain is generally the only product sold outside the cooperative association. Since specific measures for these variables could not be observed, proxy variables were identified which would indicate these price variables. The price available to the cooperative is commonly

indicated by the markets available and used by the association for its grain. Variables identified as proxies for this include: the percentage of grain shipped to the market via unit train shipment; the distance in mileage to alternative markets; percentage of grain shipped to the alternative markets; etc.

The final variable,  $\bar{K}$ , is the total amount of capital employed by the cooperative association. This variable was measured in the balance sheet as total liabilities.

Many variables were observed in the firm which would be indicative of several of the general categories constructed here. One such variable would be unit train shipment. Unit train shipment represents the ability to utilize market outlets available to the cooperative, most commonly the gulf market. When shipping grain to the Gulf of Mexico for export, it is common to ship several grain cars in one train to obtain better shipping rates. Being able to use unit train shipping often allows the cooperative to consider the gulf market for its grain, otherwise it is too expensive to ship the grain that distance.

Unit train shipment is also an indication of expenses incurred in the operation. To be able to ship unit trains of grain requires an investment in facilities which will change the average cost of operation. This dual role of unit train shipment as an indicator for variables in the model requires us to analyze both roles in the analysis of the results.

The exogenous variables used in the analysis could be represented by a matrix  $Z$ , where  $Z_{i\ell}$  is the observation for the  $\ell$ -th exogenous

variable in firm  $i$ . The variables for prices of feed and fertilizer can be represented by  $P_{F_1}$ ,  $P_{F_2}$ ,  $P_{F_3}$ , and  $P_{A_1}$ , where  $P_{F_1}$  is the vector of cross-sectional prices for 40% hog ration,  $P_{F_2}$  is the vector of cross-sectional prices for 35% cattle ration,  $P_{F_3}$  is the vector of cross-sectional prices for 50% cattle ration, and  $P_{A_1}$  is the vector of cross-sectional prices for anhydrous ammonia. The models can be represented in matrix form as;

$$P_i = ZB_i + \varepsilon_i, \quad (4.15)$$

where  $i$  represents the different products ( $i = F_1, F_2, F_3$ , and  $A_1$ ),  $B_i$  is the vector of coefficients for the exogenous variables in the  $i$ -th model, and  $\varepsilon_i$  is the vector of error components for the  $i$ -th model. Applying ordinary least squares to these models will derive estimates of the  $B_i$ , i.e.;

$$\hat{B}_i = (Z'Z)^{-1} Z'P_i \quad (4.16)$$

where 4.16 is the normal equation for the ordinary least squares procedure.

#### Grain models

The models hypothesized for the cross-sectional and time series information for corn and soybeans were presented in Equations 4.3 and 4.4. Because of the error structure in the models presented, a generalized least squares regression procedure must be used in the estimation process.

The dependent variable chosen for analysis in these models was the weekly Thursday closing basis level determined for each firm from January 6, 1977, through May 31, 1979. The hypothesized model states that the values of these observations are dependent on exogenous time series elements ( $X_{ikt}$ ) and exogenous cross-sectional elements ( $Z_{i0}$ ).

The exogenous cross-sectional variables in the analysis are the same variables which were proposed in the feed and fertilizer models. Variables were constructed which would represent the different general classes of exogenous variables in the general reduced form Equation 4.1.

The exogenous time series variables used in the analysis included a utilization-of-facilities variable and time series factors such as trend and seasonal elements. The utilization variable was determined from the grain storage records of the cooperative associations. Total grain in storage at the end of each month was measured. Each stored grain level was divided by total storage available to determine the percentage of storage utilization at the end of each month. Each utilization level was matched with the basis levels for the same month.

The other time series factors used in the analysis were trend, seasonal dummy variables, and trigonometric functions to account for seasonal effects. These time series factors along with the utilization-of-facilities time series represented the exogenous time series variables,  $X_{ikt}$ , in the analysis.

The models for soybean and corn basis levels can be rewritten as;



$$BB_{it} = [A_o + \sum_{\ell=1}^L \gamma_{\ell} Z_{i\ell}] + \sum_{k=1}^K [A_k + \sum_{\ell=1}^L (\gamma)_{k\ell} Z_{i\ell}] X_{ikt} + \epsilon_{it} \quad (4.17)$$

and;

$$BC_{it} = [B_o + \sum_{\ell=1}^L \rho_{\ell} Z_{i\ell}] + \sum_{k=1}^K [B_k + \sum_{\ell=1}^L (\rho)_{k\ell} Z_{i\ell}] X_{ikt} + \eta_{it} \quad (4.18)$$

Consolidating the terms in brackets, we can redefine the equations for each firm as;

$$BB_{it} = D_{io} + \sum_{k=1}^K D_{ik} X_{ikt} + \epsilon_{it} \quad (4.19)$$

and

$$BC_{it} = G_{io} + \sum_{k=1}^K G_{ik} X_{ikt} + \eta_{it} \quad (4.20)$$

Realizing these models are time series models, a time series analysis can be done for each firm. The error structure for each firm was assumed to have cross-sectional independence (condition 4.10). It is assumed that the error structure of all firms will be of the same type, but that heteroscedasticity is possible.

It is important to specify an appropriate error structure so that the best linear unbiased estimates of the coefficients of the time series models are obtained. In time series data the successive residuals in a model tend to be highly correlated. This correlation is known as serial correlation or autocorrelation.

Serial correlation in time series models is often assumed to

be either an autoregressive process, a moving-average process, or a mixed model of the two. The error structure of the models in Equations 4.19 and 4.20 will be assumed to be the same across firms. Heteroschedasticity may exist, but the basic structure of the residuals will be the same. In time series data, the most common type of error structure assumed in most analyses is an autoregressive process. An autoregressive process in the error structure can be shown as;

$$\epsilon_{it} = \theta_1 \epsilon_{i,t-1} + \theta_2 \epsilon_{i,t-2} + \dots + \theta_k \epsilon_{i,t-k} + u_{it} \quad (4.21)$$

where  $\epsilon_{it}$  is the observed error of the  $i$ -th firm in the  $t$ -th time period,  $\theta_1$  through  $\theta_k$  are the coefficients of the autoregressive process of order  $k$ , and  $u_{it}$  is the random error of the  $i$ -th firm in the  $t$ -th time period.

In examining the error structure of the time series models in Equations 4.19 and 4.20, a typical firm was chosen for the analysis to determine the structure. In the examination process an autoregressive process of the residuals was assumed of the form in Equation 4.21. The examination dealt with specifying the order of the process, i.e., the number of time periods,  $k$ , which were involved in the process.

The first phase for the error structure analysis was the application of ordinary least squares for the models in Equation 4.19 and 4.20 for the typical firm. The resulting estimates of the regression coefficients were unbiased and consistent, and were used in the second phase of the error structure analysis to calculate the ordinary least squares

regression residuals  $e_{it}$ . From this series of calculated regression residuals, we can specify estimates of the coefficients in the error models for each firm as specified in Equation 4.21.

Given an appropriate error structure, an analysis of the models in Equations 4.19 and 4.20 can be done for each firm. If, for firm  $i$ , we specify the covariance matrix as  $\Omega_{1i}$  for the soybean basis model and  $\Omega_{2i}$  for the corn basis model, the vector of soybean basis levels as  $BB_i$ , the vector of corn basis levels as  $BC_i$ , and the matrix of time series elements as  $X_i$ , then using generalized least squares procedure will yield the Aitken's estimator<sup>1</sup> of,

$$\tilde{D}_i = (X_i' \Omega_{1i}^{-1} X_i)^{-1} (X_i' \Omega_{1i}^{-1} BB_i) \quad (4.22)$$

and

$$\tilde{G}_i = (X_i' \Omega_{2i}^{-1} X_i)^{-1} (X_i' \Omega_{2i}^{-1} BC_i). \quad (4.23)$$

$\tilde{D}_i$  and  $\tilde{G}_i$  would be the best linear unbiased estimators<sup>1</sup> of  $D_i$  and  $G_i$ . The asymptotic variance-covariance matrix of  $\tilde{D}_i$  is,

$$\text{Asympt. Var-Cov } (\tilde{D}_i) = (X_i' \Omega_{1i}^{-1} X_i)^{-1} \quad (4.24)$$

and  $\tilde{G}_i$  is

$$\text{Asympt. Var-Cov } (\tilde{G}_i) = (X_i' \Omega_{2i}^{-1} X_i)^{-1}. \quad (4.25)$$

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<sup>1</sup>See J. Kmenta (30, p. 504).

After computing the estimates of  $\tilde{D}_i$  and  $\tilde{G}_i$ , estimates of the coefficients  $A_o$ ,  $\gamma_\ell (\ell \in Z)$ ,  $A_k (k \in X)$ , and  $(A\gamma)_{k\ell}$  for the soybean basis model and  $B_o$ ,  $\rho_\ell (\ell \in Z)$ ,  $B_k (k \in X)$ , and  $(B\rho)_{k\ell}$  for the corn basis model can be found. To do this, it must be realized that the estimated coefficients in the  $\tilde{D}_i$  and  $\tilde{G}_i$  vectors are consistent, asymptotically efficient, and asymptotically normal, with the asymptotic variance-covariance matrices given in Equations 4.24 and 4.25. Noting that they are asymptotically normal, we can use the estimates from  $\tilde{D}_i$  and  $\tilde{G}_i$  to regress on the cross-sectional characteristics  $(Z_{i\ell})$  to derive the estimates for the coefficients in the overall model given in Equations 4.3 and 4.4. The regression models we would fit would be,

$$\tilde{D}_{ik} = A_k + \sum_{\ell=1}^L (A\gamma)_{k\ell} Z_{i\ell} + e_{ik} \quad (4.26)$$

$$\tilde{G}_{ik} = B_k + \sum_{\ell=1}^L (B\rho)_{k\ell} Z_{i\ell} + w_{ik} \quad (4.27)$$

where  $\tilde{D}_{ik}$  is the estimated coefficient for the k-th time series variable for the i-th firm in the soybean model,  $Z_{i\ell}$  is the  $\ell$ -th exogenous cross-sectional variable for the i-th firm, and  $e_{ik}$  is the error in the k-th coefficient for the soybean model in the i-th firm.  $\tilde{G}_{ik}$ , and  $w_{ik}$  are the corresponding variables for the corn basis model.

In fitting the models in Equations 4.26 and 4.27, we will be able to define unbiased estimates of the coefficients in the overall models defined in Equations 4.3 and 4.4. Given an appropriate specification of the error structure we can define the coefficients in Equations 4.3 and 4.4 from Equations 4.26 and 4.27.

Realizing the distribution of the  $\tilde{D}_i$  and  $\tilde{G}_i$  is not homoscedastic across firms indicates that generalized least squares should be used to get the "best" estimates of the coefficients in Equations 4.26 and 4.27. To apply generalized least squares the variables must be transformed to represent a normal distribution across firms. If the variance-covariance matrices of the errors are represented by  $\Omega_D$  and  $\Omega_G$ , the Aitken's estimates can be derived, i.e.;

$$\tilde{A}\gamma = (Z'\Omega_D^{-1}Z)^{-1} (Z'\Omega_D^{-1}\tilde{D}) \quad (4.28)$$

$$\tilde{B}\rho = (Z'\Omega_G^{-1}Z)^{-1} (Z'\Omega_G^{-1}\tilde{G}) \quad (4.29)$$

where  $\tilde{A}\gamma$  and  $\tilde{B}\rho$  are the vectors of estimated coefficients in Equations 4.26 and 4.27,  $Z$  is the matrix of exogenous variables in Equations 4.26 and 4.27, and  $\tilde{D}$  and  $\tilde{G}$  are the vectors of estimated coefficients from Equations 4.22 and 4.23. The variance-covariance matrix of  $\tilde{A}\gamma$  and  $\tilde{B}\rho$  are;

$$\text{var-cov}(\tilde{A}\gamma) = (Z'\Omega_D^{-1}Z)^{-1} \quad (4.30)$$

$$\text{var-cov}(\tilde{B}\rho) = (Z'\Omega_G^{-1}Z)^{-1}. \quad (4.31)$$

If ordinary least squares is applied to Equations 4.26 and 4.27 then the normal Equations for  $\tilde{A}\gamma$  and  $\tilde{B}\rho$  can be derived, i.e.;

$$\tilde{A}\gamma = (Z'Z)^{-1}(Z'\tilde{D}) \quad (4.32)$$

$$\tilde{B}\rho = (Z'Z)^{-1}(Z'\tilde{G}). \quad (4.33)$$

where  $\tilde{A}\gamma$  and  $\tilde{B}\rho$  are the estimates of the coefficients in Equations 4.26

and 4.27 using ordinary least squares procedures. The variance-covariance matrix for  $\tilde{A}\gamma$  can be derived as;

$$\text{var-cov}(\hat{A}\gamma) = E[(\hat{A}\gamma - A\gamma)(\hat{A}\gamma - A\gamma)'] \quad (4.34)$$

where  $A\gamma$  is the vector of actual values of the coefficients. Equation 4.34 can be rewritten as;

$$\text{var-cov}(\hat{A}\gamma) = E[(Z'Z)^{-1}Z'\Omega_D Z(Z'Z)^{-1}]. \quad (4.35)$$

or;

$$\text{var-cov}(\hat{A}\gamma) = (Z'Z)^{-1}Z'\Omega_D Z(Z'Z)^{-1}. \quad (4.36)$$

The variance-covariance matrix for  $\hat{B}\rho$  is;

$$\text{var-cov}(\hat{B}\rho) = (Z'Z)^{-1}Z'\Omega_G Z(Z'Z)^{-1}. \quad (4.37)$$

From Equations 4.30 and 4.31 and Equations 4.36 and 4.37, we can note;<sup>1</sup>

$$\text{var-cov}(\hat{A}\gamma) \geq \text{var-cov}(\tilde{A}\gamma) \quad (4.38)$$

$$\text{var-cov}(\hat{B}\rho) \geq \text{var-cov}(\tilde{B}\rho). \quad (4.39)$$

Equations 4.38 and 4.39 point out the disadvantages of using ordinary least squares for Equations 4.26 and 4.27, i.e., the ordinary least squares procedure does not yield the "best" estimator since the variance of the ordinary least squares estimator is biased. The ordinary least squares estimator is, however, unbiased and is "like"<sup>2</sup>

<sup>1</sup>See J. Johnston (29b, p. 210).

<sup>2</sup>W. Fuller. Taken from class notes developed in Econometric Methods.

the generalized least squares estimator for large  $n$  if  $Z$  satisfies;

$$\lim_{n \rightarrow \infty} n^{-1} Z'Z = D \quad |D| \neq 0 \quad (4.40)$$

and;

$$\lim_{n \rightarrow \infty} n^{-1} Z' \Omega^{-1} Z = H \quad (4.41)$$

$\hat{A}\gamma$  and  $\hat{B}\rho$  were the estimators chosen for estimating the coefficients of Equations 4.3 and 4.4. These ordinary least squares estimators were chosen for this phase of the empirical procedure since it is the final step in the regression analysis and it does result in unbiased estimates. The estimated variance of each coefficient will be equal to or larger than the variance with generalized least squares procedures. We can, however, use the results here to specify and discuss the results of the models in Equations 4.3 and 4.4. Generalized least squares would yield better estimates, but time and monetary considerations dictated the use of  $\hat{A}\gamma$  and  $\hat{B}\rho$ .

## CHAPTER V. COOPERATIVE PRICING AND PRODUCTION

## SUB-MODEL RESULTS

## Feed and Fertilizer Models

The models for feed and fertilizer were stated in Equation 4.11. The results were obtained by applying ordinary least squares to the data. The standard ordinary least squares assumptions were given in conditions 4.12, 4.13, and 4.14.

The equation which best fit the anhydrous ammonia fertilizer model was;

$$\begin{aligned} \hat{PA}_{1i} = & 188.997 - 1.827913 (PCAP)_i + 3.213252 (ECAP)_i \\ & (5.775) (0.965038) (1.716101) \\ & -3.420716 (SERV)_i + 18.195844 (LEV)_i - 3.165396 (CCON)_i \\ & (1.215927) (8.882489) (1.144392) \\ & -0.767206 (\bar{K})_i. \\ & (0.183946) \end{aligned}$$

$\hat{PA}_{1i}$  is the estimated price for anhydrous ammonia fertilizer for the  $i$ -th firm.  $(PCAP)_i$  is the total in-plant storage capacity in hundreds of tons for anhydrous ammonia for the  $i$ -th firm.  $(ECAP)_i$  is the total storage capacity in hundreds of tons which is available in "nurse" tanks in the  $i$ -th firm. "Nurse" tanks are used with an applicator to apply anhydrous ammonia fertilizer in the fields. Anhydrous ammonia fertilizer can be stored in these "nurse" tanks, i.e., the "nurse" tanks become another type of storage for the cooperative.  $(SERV)_i$  is the total charge (in \$10/ton units) for delivery and application of anhydrous ammonia.  $(LEV)_i$  is the leverage ratio (debt/total liabilities)



of the  $i$ -th firm.  $(CCON)_i$  is the percentage of total grain production which is corn in the county of the home office of the  $i$ -th firm, divided by 10.0.  $(\bar{K})_i$  is the total amount of capital (millions of dollars) which is employed by the  $i$ -th cooperative and was measured as total liabilities. These variables were chosen from a set of exogenous variables specified in the general reduced form Equation 4.2. The estimated coefficients for the variables are stated with the estimated standard deviations of the coefficients in parentheses below. The estimated coefficients for the intercept, SERV, CCON and  $\bar{K}$  were found to be statistically significant at a level of 99 percent. The estimated coefficient for LEV was statistically significant at a level of 95 percent and the estimated coefficients for PCAP and ECAP were statistically significant at the level of 90 percent. The mean square error was 32.4048 and the  $R^2$  was 53.30. The F-statistic was 10.46, which was significant at a level of 99 percent significance.

The significant exogenous variables are listed in Table 2 with their high, low, and average values. Other exogenous variables were fit in the analysis but were either not statistically significant or collinear with the variables discussed before, which yielded several insignificant variables (the new variable and one or more of the variables in Table 2). Those exogenous variables which were fit and considered insignificant included: a) total storage capacity of anhydrous ammonia (PCAP + ECAP); b) average farm size in acres in the county of the firm's home office; c) working capital in dollars (total

Table 2. Exogenous variables for the anhydrous ammonia model

Variable	High	Low	Average
PCAP	6.1500	0.0000	1.5344
ECAP	3.1200	0.0000	0.9087
SERV	2.8000	0.0000	1.1096
LEV	0.5368	0.0493	0.2090
CCON	5.5556	2.0755	4.4334
$\bar{K}$	25.5223	0.9523	6.0093

current assets less total current liabilities on May 31, 1979); d) the percentage of patronage dividends paid in cash, s; e) the number of years deferred patronage dividends are deferred,  $\tau$ , and the dummy variable TD for indicating those firms who pay out the deferred portion of patronage dividends in any method other than oldest first; f) total corn production in the county of the firm's home office; g) the total dollars of operating expense which can be allocated to agronomy according to dollar sales; h) the average dollars of operating expense for agronomy per dollar of agronomy sales; and i) the simple average dividend rate on anhydrous ammonia fertilizer for the three most recent fiscal years.

The variable PCAP represents an exogenous variable from the general exogenous variable of  $Q_{FC}$  since it represents one of the fixed assets owned by the cooperative. It also may be correlated with the general exogenous variable  $P_V$ . This correlation is because the type of

facilities may dictate the types of variable inputs which have to be purchased to produce the anhydrous ammonia. The sign on the coefficient for in-plant storage is negative, meaning with all other things equal, the higher the level of in-plant storage capacity in the cooperative the lower is the price charged to the patrons. This variable may indicate that economies of scale exist in the plant capacity for anhydrous ammonia.

The exogenous variable ECAP, total equipment storage capacity in tons, represents the general exogenous variables of  $Q_{FC}$  and  $P_V$ . ECAP is representative of the general variable  $Q_{FC}$  since equipment storage indicates one of the fixed factors of production available to the cooperative. It is indicative of the general variable  $P_V$  since it may also dictate the type of variable inputs the cooperative has to purchase to produce anhydrous ammonia. The sign of the coefficient for equipment storage is positive which may be interpreted as meaning with all other things equal, the larger the level of equipment storage capacity, the higher is the price of anhydrous ammonia.

The major reason in the difference in signs between PCAP and ECAP is the type of variable inputs needed to produce anhydrous ammonia with the defined variable. By adding in-plant storage, the firm is likely to gain some economies of size in the facility. In order to add equipment storage, the firm must increase the number of units which are able to store the anhydrous ammonia. Equipment storage comes in the form of "nurse" tanks. Increasing the number of units of "nurse" tanks increases the amount of labor required to handle the fertilizer and

maintain the equipment. With everything else equal, this causes the average cost of producing and selling anhydrous ammonia to increase leading to the positive coefficient for ECAP.

The exogenous variable SERV is the total service charge for delivery and application of anhydrous ammonia. The sign on the coefficient for SERV is negative which may be interpreted as meaning with all other things being equal, the higher is the service charge for delivery and application of anhydrous ammonia the lower is the price of anhydrous ammonia. Service charges are reimbursements from the patrons to the cooperative for the cost of performing the services. If service charges represented only the cost of performing the service then SERV would be representative of the general variable  $P_V$ . There also exists the possibility of a specification error by including this variable in the analysis. It may be argued that delivery and application of anhydrous ammonia is a product produced by the cooperative and that SERV represents the price of this product. If delivery and application is a product, then complementarity is shown in the sign of the coefficient for this variable. The two products, service and anhydrous ammonia, complement each other. The lower the price charged for service, the higher is the price for anhydrous ammonia to compensate for the lower service charge.

The exogenous variable LEV, leverage ratio, is indicative of the general variable  $P_V$ . The leverage ratio is one of the determinants of the marginal interest cost for debt in the cooperative. The higher is the leverage ratio, the higher is the amount of risk assumed by the

lenders. In order to assume more risk, most lending institutions charge a higher rate of interest to compensate for the higher level of risk. Interest is a cost of obtaining debt, therefore, interest represents a cost of a variable input (debt) required to produce anhydrous ammonia. The positive correlation between the leverage ratio and interest cost indicates a positive correlation between the leverage ratio and the price of one variable input of production. The sign of the coefficient for leverage ratio is positive which may be interpreted as meaning with all other things equal, the higher is the leverage ratio the higher is the price of anhydrous ammonia.

The exogenous variable CCON, concentration of corn production in the county of the firm's home office, is indicative of the general variable  $P_V$ . The higher concentration of corn production could be associated with a lower cost of marketing the anhydrous ammonia. The lower cost of marketing could come with savings in advertising and delivery costs. Any firm which delivers the anhydrous would have a shorter average hauling distance for delivery of a fixed amount of anhydrous ammonia in a county with a high concentration of corn production. The market for anhydrous ammonia is represented by the production of corn since anhydrous ammonia is used primarily as a fertilizer for corn in Iowa. Advertising costs could be lowered, with other things being the same, since the general level of demand for anhydrous would be higher and less advertising would be needed to gain a given level of sales in comparison with a county with less corn concentration. The sign of the coefficient for corn concentration

is negative which may be interpreted as meaning with all other things equal, the higher is the concentration of corn production in the county the lower is the price for anhydrous ammonia.

The exogenous variable  $\bar{K}$ , the total capital employed, is the general variable  $\bar{K}$ . The sign of the coefficient for total capital employed is negative which may be interpreted as meaning with all other things being equal, the higher is the amount of total capital employed the lower is the price for anhydrous ammonia. Capital cannot be interpreted as being the only variable which causes the affect indicated by its coefficient. Capital will be correlated with several factors in the firm which may have more direct influence on the price of anhydrous ammonia than does capital. The coefficient for capital indicates the net influence of these variables on the price. Another general variable which would be correlated with capital would be  $Q_{FC}$ , i.e., the amount of capital could be correlated with the general level of all the fixed factors of production.

The equation which best fit the feed model for the 40 percent hog ration was;

$$\begin{aligned} \hat{P}_{Fl_i} = & 270.171 + 23.428045 (LEV)_i + 0.019290 (HENS)_i \\ & (5.705) \quad (16.009236) \quad (0.008738) \\ & -0.753568 (\bar{K})_i \\ & (0.336771) \end{aligned}$$

$\hat{P}_{Fl_i}$  is the estimated price for the 40 percent hog ration for the i-th firm.  $(LEV)_i$  is the leverage ratio which was defined in the anhydrous ammonia model (debt/total liabilities) for the i-th firm.  $(HENS)_i$  is

the variable measured as the thousands of laying hens in the county of the i-th firm's home office.  $(\bar{K})_i$  is the total capital employed (millions of dollars) in the i-th firm and is the same variable as in the anhydrous ammonia model. The estimated coefficients of the variables are stated with the estimated standard deviations of the coefficients in parentheses below. The coefficients for the exogenous variables HENS and  $\bar{K}$  were found to be statistically significant at a level of 95 percent significance and the coefficient for LEV was found to be statistically significant at a level of 80 percent. The mean square error was 160.0529 and the  $R^2$  was 16.84 percent. The F-ratio for the model was 4.12 which was significant at a level of 95 percent significance. The exogenous variables are listed in Table 3 with the high, low, and average values.

The exogenous variable LEV, the leverage ratio, is again indicative of the general variable  $P_v$ . As shown in the model for anhydrous ammonia, the higher leverage ratio generally indicates a higher marginal interest cost. The sign on the coefficient for leverage ratio is positive which may be interpreted as meaning with all other things equal, the larger the leverage ratio the higher is the price for the hog ration.

The exogenous variable HENS, the number of the laying hens in the county of the firm's home office, is also indicative of the general variable  $P_v$ . Many inputs used for producing chicken feed are also used in producing 40 percent hog ration. The larger the number of laying hens the higher is the general demand for the inputs of production

Table 3. Exogenous variables for the 40 percent hog ration model

Variable	High	Low	Average
LEV	0.5368	0.0493	0.2090
HENS	840.0000	1.0000	113.0147
$\bar{K}$	25.5223	0.9523	6.0093

for hog ration. This generally indicates a higher price level for the variable inputs with all other things being equal. The sign of the coefficient for laying hens is positive which may be interpreted as meaning with all other things being equal, the larger the number of laying hens the higher is the price for the hog ration.

The exogenous variable  $\bar{K}$ , total capital employed, is the general variable  $\bar{K}$ . The sign of the coefficient for the total amount of capital employed is negative which may be interpreted as meaning with all other things being equal, the higher is the amount of total capital employed the lower is the price for the hog ration. The amount of capital will again be correlated with several factors in this model also. It is indicative of the scale of the operation and also of the general level of all fixed factors of production ( $Q_{FC}$ ). The effect indicated by the coefficient should not be interpreted as being solely dependent on the amount of capital, but on the amount of capital and the other factors correlated with capital.

Other exogenous variables were used in the regression analysis for this model, but the variables were either not statistically significant



or were collinear with the variables discussed before and listed in Table 3. The exogenous variables which were used in the regression analysis and considered insignificant included: a) dollars of feed sales in the month of May, 1979; b) number of feed manufacturing plants; c) number of pigs farrowed from December, 1979, to May, 1980, in the county of the firm's home office; d) number of grain fed cattle marketed in 1979 from the county of the firm's home office; e) the percentage of total grain production which was corn in the county of the firm's home office; f) feed manufacturing capacity in tons for the firm; g) delivery capacity of feed in tons for the firm; h) the simple average dividend rate on feed for the three most recent fiscal years; i) total storage capacity in bushels for grain; j) total working capital (total current assets less total current liabilities on May 31, 1979); k) average farm size in acres in the county of the firm's home office; l) proportion of patronage dividends paid in cash(s); m) the number of years deferred patronage dividends are deferred,  $\tau$ , and the dummy variable TD for indicating those firms who pay out the deferred portion of patronage dividends in any method other than the oldest first; and n) the average dollars of operating expense for feed per dollar of feed sales.

The equations for 35 percent cattle ration and 50 percent cattle ration had no coefficients which were found statistically significant. It was found that the best estimate of the prices for these variables is the mean of each variable.

The pricing mechanism for feeds is what is believed to cause the

models to be inadequate. Most local cooperatives purchase their feed from a regional cooperative for marketing to their patrons. The regional cooperative supplies the cooperatives with a suggested retail price list. Many cooperatives use the suggested retail price list of the regional cooperatives as a base price list for their feeds. This type of pricing mechanism should cause a small standard deviation in the prices the cooperatives charge for feed. The large standard deviation in the feed prices reported in the sample suggests that a large portion of the standard deviation is caused by measurement error. The regional cooperatives send out suggested retail price sheets weekly. The price sheet generally lists a price for a feed in a certain form (e.g., 1 ton, bagged, meal, undelivered) i.e., discounts or additional charges are made to conform to the type of feed the sample price was for. For the 40 percent hog ration, the price asked for was a one ton bulk load of meal, undelivered, for June 14, 1979. To give the correct price the cooperative needed to get the price sheet for the week of June 14, 1979, and make the proper discounts and additional charges to arrive at the sample price. The large standard deviation in the prices of the feeds is assumed to be caused by the measurement of the feed prices. The cooperatives had the possibility of using the wrong price list or not making the appropriate discounts and additional charges. These errors in measurement would cause a large standard deviation in the prices of feeds, which makes empirical analysis of the model impossible.

## Grain Models

The estimation of the corn and soybean basis models in Equations 4.3 and 4.4 involved a three step estimation process. The first step in the analysis involved the specification of an error structure for the time series in each of the firms. The structure of the errors in each of the time series was assumed to be the same, but allowed for heteroschedasticity to be present across firms. The process followed in this step was the examination of the error structure of a typical firm selected at random. Once the basic error structure was defined for this typical firm, the same structure was used in the other firms.

The second step in the analysis of the grain models was the estimation of a time series equation for each of the individual firms. In this step of the analysis the error structure determined for the typical firm was assumed for the remaining firms. The exogenous variables included in this analysis were utilization-or-facilities, trend, monthly and quarterly seasonal dummy variables, and trigonometric functions to account for seasonal affects.

The third step of the analysis involved the regression of the coefficients derived in step two on the cross-sectional independent variables of the firms. From this regression the coefficients of the models specified in Equations 4.3 and 4.4 were determined.

### Error specification

Specifying the error structure involved the analysis of the time series model for a typical firm chosen at random from the sample of firms. In the analysis of the error structure, an autoregressive process of the residuals was assumed of the form stated in Equation 4.21. The examination dealt with specifying the order of the autoregressive process, i.e., the number of time periods,  $k$ , which were involved in the process.

Examining the error structure of the firm chosen at random revealed that an autoregressive error structure of order 2 was the appropriate error structure for both the corn and soybean basis models, i.e., the error structures could be modeled as;

$$\epsilon_{it} = \theta_{1i}\epsilon_{i,t-1} + \theta_{2i}\epsilon_{i,t-2} + u_{it} \quad (5.3)$$

$$\eta_{it} = \omega_{1i}\eta_{i,t-1} + \omega_{2i}\eta_{i,t-2} + \mu_{it} \quad (5.4)$$

$\theta_{1i}$  and  $\theta_{2i}$  are the coefficients of the error structure for the soybean basis model in firm  $i$  and  $\omega_{1i}$  and  $\omega_{2i}$  are the coefficients of the error structure for the corn basis model in firm  $i$ .  $u_{it}$  and  $\mu_{it}$  are the random error elements for each of the respective models.

### Estimation of the time series coefficients

The models to be estimated in this step of the analysis were stated in Equations 4.19 and 4.20. First, the estimated coefficients of the error structures in Equations 5.3 and 5.4 were estimated for each firm. Second, the Aitken's estimators derived in Equations 4.22

and 4.23 were applied to the models.

Only sixty-one firms could supply complete information for the utilization-of-facilities variable. Because of the lack of information from six firms, two models were specified for the corn and soybean basis models. The first model contained the utilization-of-facilities variable, trend, and seasonal components. The second model contained only the trend and seasonal components. The number of observations for most firms was 126. Some firms had some missing observations, yielding fewer than 126 observations, yet enough for estimation of the models. The number of firms used for the analysis in the first model was sixty-one while all sixty-eight were used in the second model.

The seasonal components for each of the models were chosen in the first step of this analysis in conjunction with the estimation of the error model coefficients. Monthly seasonal dummy variables, quarterly seasonal dummy variables, and various trigonometric functions were tested. The seasonal variables chosen for the analysis of the time series models were monthly seasonal dummy variables and a trigonometric function. The specific trigonometric function used was;

$$\text{Trig} = t * \text{SIN}\left(\frac{2\pi w}{52}\right). \quad (5.5)$$

$t$  is the time period of the observation (1 through 126) and  $w$  is the chronological week of the year that the observation appears in (1 through 52). The sine function allows for a trigonometric seasonal factor with the production period being one year. Multiplying the sine function by the trend element ( $t$ ) allows for a multiplicative

seasonal component, meaning that the seasonal factor from the sine function is becoming larger in absolute value throughout the time period.

The results of the analysis of the four time series models are summarized in Tables 4, 5, 6, and 7. The tables list the correlation coefficients of the estimated coefficients of the time series models and the means of the coefficients of the four time series models which were derived in Equations 4.19 and 4.20.

Some interesting notes may be made about the results derived in this step of the analysis. We defined the "basis" as the cash price the elevator bids for the grain less the Chicago Board of Trade closing bid for the nearby option. This implies that the more positive is the basis level the higher is the cash price bid to the patron for a given Chicago Board of Trade price. The most positive value of the basis level gives the patron the highest price for his grain. From the average values of the coefficients in Tables 4, 5, 6, and 7, the highest basis level, on the average, appears in February for the soybean basis model including storage utilization and in August in the soybean basis model excluding storage utilization. The highest basis level for corn appeared in August for the model containing storage utilization and in February for the model excluding storage utilization. The lowest basis level for soybeans appeared in September for the model containing storage utilization and in March for the model excluding storage utilization. The lowest basis level for corn appears in March for the model containing storage utilization and in September for the

Table 4. Correlations and means of coefficients for the soybean basis model, including storage utilization (the body of the table contains the correlations and the means are listed at the bottom)

Variable	Inter-cept	Storage utilization	Trend	Jan.	Feb.	March	April	May
Inter-cept		-0.84895	-0.39696	0.69663	0.48260	0.17325	0.06498	-0.48830
Storage utilization			0.04953	-0.80575	-0.61949	-0.27711	-0.10878	0.55036
Trend				-0.07768	0.18414	0.26968	0.16643	0.07816
Jan.					0.78377	0.39585	0.22435	-0.37619
Feb.						0.79670	0.63209	0.04120
March							0.81356	0.39201
April								0.58653
May								
June								
July								
August								
Sep.								
Oct.								
Nov.								
Dec.								
Trig.								
MEANS	-27.9045	-7.2855	-0.0701	4.9348	5.5948	-0.2916	0.6505	2.2133

June	July	August	Sep.	Oct.	Nov.	Dec.	Trig.
-0.76106	-0.81960	-0.73790	-0.38990	0.76819	0.69487	0.69084	0.06118
0.81341	0.89530	0.84245	0.44928	-0.79482	-0.74711	-0.63547	0.06402
0.94450	0.15562	-0.00954	-0.22763	-0.14541	-0.04298	-0.21643	-0.29102
-0.75547	-0.89632	-0.90822	-0.48963	0.63409	0.56012	0.59745	-0.25144
-0.42194	-0.70886	-0.86640	-0.77660	0.26362	0.20926	0.28836	-0.65338
0.01938	-0.34699	-0.56923	-0.81735	-0.16083	-0.20131	-0.00545	-0.80154
0.24786	-0.19476	-0.41251	-0.73656	-0.33957	-0.30198	-0.24648	-0.82866
0.72930	0.38666	0.24055	-0.28240	-0.82396	-0.73255	-0.48884	-0.57334
	0.82022	0.65512	0.13898	-0.88698	-0.86059	-0.71772	-0.23437
		0.81980	0.46947	-0.71731	-0.70683	-0.67628	0.17397
			0.70165	-0.59323	-0.55519	-0.57732	0.43998
				-0.05211	-0.09485	-0.33208	0.68225
					0.85974	0.63110	0.34064
						0.65924	0.30602
							0.17772
0.0444	-2.5492	-2.0250	-9.6917	-5.5130	4.7758	1.9568	-0.0255



Table 5. Correlations and means of coefficients for the soybean basis model, excluding storage utilization (the body of the table contains the correlations and the means are listed at the bottom)

Variable	Intercept	Trend	Jan.	Feb.	March	April	May	June
Intercept		-0.79653	-0.57080	-0.77974	-0.71376	-0.48184	-0.74118	-0.44190
Trend			0.42977	0.75089	0.73276	0.59817	0.68486	0.31925
Jan.				0.68528	0.45178	0.48969	0.38740	0.09335
Feb.					0.85151	0.70811	0.76570	0.33495
March						0.75620	0.85804	0.42198
April							0.70371	0.45816
May								0.45576
June								
July								
Aug.								
Sep.								
Oct.								
Nov.								
Dec.								
Trig.								
MEANS	-24.9249	-0.2393	-7.1321	-10.9968	-21.7780	-15.5953	-5.9555	1.9903

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July	Aug.	Sep.	Oct.	Nov.	Dec.	Trig.
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0.05017	0.34458	0.68502	0.76620	0.72369	0.76800	0.73447
-0.34871	-0.24427	-0.60933	-0.72051	-0.67920	-0.74052	-0.73941
-0.28420	-0.64893	-0.49021	-0.45436	-0.54306	-0.38267	-0.56487
-0.21903	-0.62248	-0.81998	-0.82573	-0.82954	-0.72021	-0.87867
-0.13621	-0.56909	-0.86924	-0.90415	-0.85673	-0.76058	-0.92040
-0.39640	-0.66802	-0.73267	-0.71323	-0.83947	-0.70304	-0.86662
-0.00976	-0.49147	-0.85134	-0.84970	-0.85581	-0.74582	-0.89739
-0.02149	-0.29813	-0.52651	-0.49830	-0.57417	-0.57925	-0.48330
	0.27675	0.10325	-0.06860	0.09543	0.02699	0.26130
		0.57963	0.42813	0.55353	0.34495	0.65968
			0.82324	0.81671	0.61746	0.89616
				0.88017	0.81442	0.87938
					0.81115	0.90551
						0.75974
17.1832	23.4702	11.1424	1.9490	0.4465	5.2762	0.0949

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Table 7. Correlations and means of coefficients for the corn basis model, excluding storage utilization (the body of the table contains the correlations and the means are listed at the bottom)

Variable	Intercept	Trend	Jan.	Feb.	March	April	May
Intercept		-0.71253	-0.66819	-0.75552	-0.77589	-0.62397	-0.60905
Trend			0.37747	0.64765	0.68207	0.56429	0.51027
Jan.				0.74527	0.62651	0.57003	0.49272
Feb.					0.92789	0.88574	0.79868
March						0.92411	0.87779
April							0.91425
May							
June							
July							
Aug.							
Sep.							
Oct.							
Nov.							
Dec.							
Trig.							
MEANS	-30.3467	-0.0816	3.7581	4.4816	-1.0691	0.4983	2.5230

June	July	Aug.	Sep.	Oct.	Nov.	Dec.	Trig.
-0.79909	-0.78333	-0.81483	-0.38682	0.83485	0.81551	0.86054	0.16328
0.88064	0.90109	0.92127	0.60566	-0.82675	-0.76913	-0.78432	0.07505
-0.10328	-0.28225	-0.16691	-0.40622	-0.04037	-0.08725	-0.20337	-0.46455
-0.77232	-0.84160	-0.90167	-0.73610	0.58457	0.47437	0.60846	-0.40409
-0.54601	-0.74809	-0.80843	-0.89924	0.28246	0.20819	0.29593	-0.68975
-0.13086	-0.39798	-0.46113	-0.80447	-0.19361	-0.23693	-0.15416	-0.87064
0.19941	-0.18858	-0.16711	-0.53390	-0.41204	-0.51974	-0.41900	-0.82437
0.67606	0.43476	0.43257	-0.11975	-0.83852	-0.89056	-0.82521	-0.66801
	0.80899	0.83239	0.39054	-0.86300	-0.82333	-0.85285	-0.10389
		0.91811	0.63428	-0.73548	-0.65303	-0.70044	0.24224
			0.74108	-0.72878	-0.66282	-0.73651	0.26700
				-0.21683	-0.16515	-0.26317	0.70934
					0.94181	0.89817	0.39315
						0.93259	0.47766
							0.33272
5.1428	20.0133	26.6046	10.9143	-3.2082	-3.8324	2.5791	0.0756

Table 6. Correlations and means of coefficients for the corn basis model, including storage utilization (the body of the table contains the correlations and the means are listed in the bottom)

Variable	Intercept	Storage utilization	Trend	Jan.	Feb.	March	April	May
Intercept	-0.86230	-0.17712	0.64967	0.46521	0.03205	-0.09436	-0.74555	
Utilization		-0.20918	-0.82093	-0.70066	-0.29679	-0.01444	0.60518	
Trend			0.18838	0.45968	0.51574	0.38149	0.25913	
Jan.				0.83178	0.49799	0.19982	-0.29416	
Feb.					0.81947	0.51260	0.03067	
March						0.72485	0.46321	
April							0.60779	
May								
June								
July								
Aug.								
Sep.								
Oct.								
Nov.								
Dec.								
Trig.								
MEANS	-33.5001	12.7315	-0.2208	-8.7154	-11.4915	-20.5661	-14.2776	-3.1628

June	July	Aug.	Sep.	Oct.	Nov.	Dec.	Trig.
0.60340	0.13934	0.24517	0.18525	0.21409	0.07356	-0.07374	0.37656
-0.30244	-0.00421	-0.32510	-0.42227	-0.32325	-0.19719	-0.11631	-0.42548
-0.52583	-0.52112	-0.45908	-0.21370	-0.18447	-0.01055	0.24379	-0.39015
-0.38713	-0.30877	-0.66463	-0.55790	-0.58603	-0.37209	-0.14536	-0.66107
-0.44838	-0.22255	-0.57360	-0.52184	-0.58244	-0.32753	-0.17434	-0.64986
-0.33515	-0.16629	-0.61774	-0.61316	-0.70981	-0.41557	-0.26473	-0.68327
-0.52334	-0.05672	-0.43022	-0.44566	0.50632	-0.22644	-0.12666	-0.56669
	0.09314	-0.29321	-0.32363	-0.36561	-0.60560	-0.52613	-0.00061
		0.39755	0.04411	0.02328	-0.10755	-0.16669	0.18355
			0.74062	0.74790	0.68599	0.41517	0.67887
				0.78957	0.71250	0.24232	0.58395
					0.83209	0.59024	0.65181
						0.65567	0.50661
							0.29766
3.2811	-1.0958	-0.2176	-8.6773	-6.8742	3.2109	0.1810	-0.0273

model excluding storage utilization.

The average value of the coefficient for storage utilization was negative in the soybean basis model, which may be interpreted as meaning with all other things being equal, the higher the level of storage utilization the lower is the basis bid to the patron for soybeans. The average value of the same coefficient in the corn basis model was positive, yielding the opposite result which is interpreted as meaning with all other things being equal, the higher the level of storage utilization the higher is the basis bid to the patrons for corn.

The differences in the coefficients between the two models for each product is caused by the addition of the storage utilization variable. The storage utilization variable tends to follow a seasonal pattern. The highest value for this variable tends to be in the late months of the year (October, November, and December) following the harvest of the two products. The value then diminishes throughout the production period as grain is shipped to final destination for use as a final product. Given the seasonal pattern of this variable, it accounts for some of the seasonal affects through multicollinearity with the seasonal elements. The collinearity of the storage utilization variable with the seasonal dummy variables is what likely caused the change in the value of the coefficients of the seasonal dummy variables when the storage utilization variable was added.

The storage utilization coefficients were significant at a level of 75 percent or more in 38 of the soybean basis time series models and 22 of the corn basis time series models. The choice of which model was

most appropriate would probably be for the model which includes the storage utilization variable. If the model which includes storage utilization is chosen as the "true" model, then we must assume that specification bias exists in the estimated coefficients of the models which exclude the storage utilization variable. If a specification error of the model exists then the estimated coefficients will be biased.<sup>1</sup> The addition of the storage utilization variable would eliminate the specification bias in these models. Since the results do not substantially support either model as being better than the other (including the storage utilization variable versus excluding the variable), the results of both models have been reported.

The mean values of the coefficients indicate some pattern in the coefficients of the seasonal dummy variables. Another indication of the seasonal pattern can be seen in the correlation coefficients of the seasonal dummy variables (Tables 4 through 7). While the means indicate the overall average of the coefficients, the correlation coefficients indicate the correlation between the coefficients within each of the cooperatives. The pattern of the correlation coefficients indicates that, on the average, a seasonal pattern exists within the firms. This is indicated by the values and signs of the correlation coefficients, e.g., the correlation coefficients in Table 3 for January dummy variable coefficients show: positive correlations with February, March,

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<sup>1</sup>See Z. Griliches (23, pp. 8-20).



and April; negative correlations with May, June, July, August, and September; and positive correlations with October, November, and December. Similar observations can be made on the other coefficients within the four models which indicate a definite seasonal pattern in the time series.

#### Estimation of the final coefficients

This step of the analysis involves the regression of the estimated time series coefficients derived in the previous step on the cross-sectional variables of the firms. The results of these regressions derive the coefficients of the models as specified in Equations 4.26 and 4.27.

The results of this step of the analysis are contained in Tables 8 through 11. A regression was fit for each time series coefficient in each of the four models. Twenty-two cross-sectional variables were fit in each regression. Ordinary least squares was used for the analysis which yielded unbiased estimates of the coefficients.

The twenty-two cross-sectional variables listed down the sides of Tables 8 through 11 represent the general exogenous variables of the reduced form Equation 4.1. The variable "turnover" represents the number of times the available storage of the cooperative is used during the year, i.e., it equals total sales of grain in bushels divided by total storage available in bushels. This variable would be representative of the general exogenous variables  $P_V$  and  $Q_{FC}$ . It would represent  $P_V$  since a higher utilization of storage facilities reduces it's

cost. It would represent  $Q_{FC}$  since lower utilization could represent over expansion of the fixed factors of production.

The variable "storage" represents the general exogenous variable  $Q_{FC}$ . Storage is calculated as the millions of bushels of grain storage capacity (all types) the cooperative has available. Storage is a fixed factor of production in the cooperative.

The variable "employees" represents the general variable  $P_V$ . Employees was computed as the total number of employees (in tens) the cooperative employs on a full-time basis. This represents a variable cost of production in the cooperative, i.e., the more employees for a given cooperative size, the higher is the expenditure on labor per unit of output.

The variable "concrete" represents the general variable  $Q_{FC}$ . Concrete was computed to be the proportion of all storage which is upright concrete storage, i.e., concrete storage divided by total storage. Concrete storage was separated out because it has generally been considered to be more efficient (in terms of unit cost of production) than other types of storage. Concrete storage is again a fixed factor of production just as "storage" was considered to be.

The variable "FAPV" represents the general variable  $Q_{FC}$ . This variable was computed as the millions of dollars of fixed assets in purchased value available to the cooperative. This variable represents the general level of fixed factors of production.

The variable "distance" represents the general variable  $P_Z$ . This variable was computed by multiplying the proportion of grain shipped to

each local market by the number of air miles to each of these markets, and then summing them. A local market was considered to be any destination other than the Gulf of Mexico. The sum was then divided by 100 to get the variable "distance". Distance is representative of the general variable  $P_Z$  since the closer the cooperative is located to the final markets, the higher will be the net price (price received less transportation cost), other things being equal.

The variables "s gulf" and "c gulf" are also representative of the general variable  $P_Z$ . S gulf and c gulf were computed to be the proportion of soybeans (former) and corn (latter) shipped directly to the Gulf of Mexico as final destination. These variables represent a product market which is generally considered to offer higher prices for corn and soybeans since they are shipped for export from the gulf.

The variable "pigs" represents the general exogenous variable  $P_Z$ . This variable was computed as the inventory of hogs and pigs on farms (in 100,000 units) in the county of the firm's home office on December 1, 1977. Pigs represent a market for corn and soybeans since both are used to produce pig feed. The larger the number of pigs, the larger is the demand for pig feed, all other things being equal.

The variable "concentration" is representative of the general variable  $P_V$ . Concentration was computed by dividing the total storage capacity of the cooperative by the total storage capacity (commercial plus private) in the county of the firm's home office. Concentration is representative of  $P_V$  since it represents the amount of competition for utilizing storage facilities. The lower is the level of concentration

the higher will be the cost of obtaining grain to utilize the storage capacity of the firm, all other things equal.

The variable "all cattle" represents the general exogenous variable  $P_Z$ . All cattle was computed as the total number of all cattle (in 100,000 units) in the county of the firm's home office on January 1, 1979. All cattle represents a market for the product since corn and soybeans are used to produce cattle feed. The larger the number of cattle, the larger is the demand for cattle feed, all other things being equal.

The variable "grain cattle" is representative of the general variable  $P_Z$ . Grain cattle was computed as the total number of cattle (in 100,000 units) in the county of the firm's home office which were grain fed for marketing in the year 1979. Grain cattle represents  $P_Z$  in the same way as pigs and all cattle. Grain cattle and all cattle were both used in the analysis since all cattle represents the general class of cattle and grain cattle represents a more specific type of cattle. The type of market differs between grain cattle and all cattle.

The variable "hens" is also representative of the general variable  $P_Z$ . This variable was computed as the total number of laying hens (in units of 1,000,000) in the county of the firm's home office. This variable also represents a market for grain in the same way as pigs, all cattle, and grain cattle.

The variable "F concen" is representative of the general variable  $P_V$ . F concen was computed by dividing the storage capacity of the firm by total commercial storage in the county of the firm's home office.

F concen represents competition for storage utilization in a manner similar to the "concentration" variable. F concen represents the direct competition between commercial storage firms for utilizing the storage.

The "bushel expense" variable is also representative of the general variable  $P_V$ . Bushel expense was computed by dividing the total expenses of the cooperative which could be allocated to the marketing of grain, by total grain sales in bushels (both for the most recent fiscal year). This variable (in units of cents per bushel) would be an indication of the general level of variable input prices,  $P_V$ .

The variable "dry 24" was computed as the price in cents per bushel the cooperative charges it's patrons for drying 24 percent moisture corn to storage level moisture, divided by ten. If dry 24 represented only the cost of drying the corn, then dry 24 would be representative of the general variable  $P_V$ . There also exists the possibility that a specification error is involved by including this variable. It may be argued that drying grain is a product produced by the cooperative and that dry 24 represents the price of this product. If drying is considered a product then complementarity may exist, as was the case in the anhydrous ammonia model with SERV, or the variable may have no effect on the price of grain.

The variable "shrink" was computed as the percent of weight deducted from grain for each percent of moisture removed in the drying process. This variable can be analyzed with interpretations similar to the dry 24 variable. If shrink represents the true loss of weight in the drying

process, then shrink is representative of the general variable  $P_V$ . If, however, we consider drying grain to be a product of the cooperative, then complementarity may exist, or the variable may have no effect on the price of grain. If drying grain is a product then a specification error has been incorporated in the model by including shrink in the analysis of grain prices.

The variables " $\bar{K}$ ", "s", " $\tau$ ", "TD", and "LEV" are the same variables as  $\bar{K}$ , s,  $\tau$ , TD, and LEV, which were all described in the feed and anhydrous ammonia models.  $\bar{K}$  represents the general variable  $\bar{K}$ ; s represents the general variable s;  $\tau$  and TD represent the general variable  $\tau$ ; and LEV represents the general variable  $P_V$ .

The variables "sdiv" and "cddiv" represent the general variable  $R_C$ . Sdiv and cdiv are calculated as a simple average of the past three years dividend rates for soybeans and corn, respectively. These variables would be indicative of the expected level of dividends of the member patrons since most members would use history as their best judge of expected rates.

The results in Tables 8 through 11 show the coefficients of the cross-sectional variables which have significant t-statistics at a level of 80 percent or more. The F-statistics indicate the significance of the models which were fit to the time series coefficients. In the regressions where the F-statistic is not significant, the best estimate of the coefficient is the mean value of the time series coefficient (located in Tables 4 through 7). The level of significance for the F-statistics which were chosen as acceptable for constructing the models

Table 8. Final coefficients for the soybean basis model, including storage utilization

Variable <sup>a</sup>	Storage utilization	Intercept	Trend	Jan.	Feb.	March	April	May
Intercept								
Turnover			-0.4951***			-19.7719*		-19.9686*
Storage	-31.8896**			5.0926**	3.6145*			
Employees								
Concrete			-0.1291*		-10.2629*	-11.3383**		
FAPV	45.416***			-6.8499***	-6.2002**	-5.2874*		
Distance								
S gulf							10.9969*	
Pigs					6.6538*	7.5907*	7.5584*	
Concentration								-6.9803*
All cattle							-21.1700*	
Grain cattle								
Hens								
F concen	181.9203*		-0.5080**	-51.7071****	-40.3943***	-34.0505**		
Bushel exp.								
Dry 24								
Shrink		-139.9452*						48.9360*
K	-16.5033**			2.8062***	2.6256***	1.9876*		
s								
t								
TD	60.0439*							
Sdiv				1.4826*	1.7188**	1.7927**		
LEV	-195.2278*				30.2219*	24.0237*		
R <sup>2</sup>	27.88	31.30	30.07	31.11	31.45	32.21	25.96	29.89
F-statistic	0.65	0.77	0.72	0.76	0.77	0.80	0.59	0.72

<sup>a</sup>The variables across the top are the time series variables and the variables down the side are the cross-sectional variables.

\* t-statistic significant at 80 percent.

\*\* t-statistic significant at 90 percent.

\*\*\* t-statistic significant at 95 percent.

\*\*\*\* t-statistic significant at 99 percent.

+ F-statistic significant at 50 percent.

June	July	Aug.	Sep.	Oct.	Nov.	Dec.	Trig.
-7.1056***	-5.4819*	-8.4252**		6.8608*			0.2390*
			12.4281*				0.1166*
7.4638**	7.3160*	11.7341***	7.7540*				
					-17.6691*		-0.1527**
							-0.0848*
							0.2388*
56.1796***	36.0051*	63.1635**	48.6044**				0.2689*
	-9.7843*						
				-89.0294*		-62.8913*	
-2.7136**	-2.4601*	-4.3882**	-2.8847*				-0.0184*
0.6563*							
13.1283*						-9.7679*	
	-1.8553*	-2.0918*					-0.0219***
	-39.4205*						
33.08	23.32	28.41	25.08	30.90	22.95	25.65	36.19
0.83	0.51	0.67	0.56	0.75	0.50	0.58	0.95 <sup>+</sup>



Table 9. Final coefficients for the soybean basis model, excluding storage utilization

Variable <sup>a</sup>	Intercept	Trend	Jan.	Feb.	March	April	May
Intercept						74.1645*	
Turnover		-0.2669*					
Storage							
Employees							
Concrete	-8.0830*	-0.1001*	-4.7152**	-8.8731***	-7.2843*		
FAPV							
Distance					9.1685**		
S gulf						8.8452*	
Pigs						7.0601*	
Concentration					-8.1832**	-7.2960*	-6.9544***
All cattle					-29.6538***	-35.2849****	-25.3183***
Grain cattle					19.4414**	24.9248***	20.6244***
Hens							
F concen			-7.9522*	-11.2815*	-17.3151*		
Bushel Exp.							
Dry 24				-4.7778*			
Shrink							
K							
s						-3.5411*	
t							
TD							
Sdiv	-1.1870*						
LEV							
R <sup>2</sup>	20.28	17.65	25.39	32.36	26.68	34.01	23.04
F-statistic	0.51	0.43	0.68	0.93 <sup>†</sup>	0.73	1.03 <sup>†</sup>	0.60

<sup>a</sup>The variables across the top are time series variables and the variables down the side are the cross-sectional variables.

\* t-statistic significant at 80 percent.

\*\* t-statistic significant at 90 percent.

\*\*\* t-statistic significant at 95 percent.

\*\*\*\* t-statistic significant at 99 percent.

<sup>†</sup> F-statistic significant at 50 percent.

<sup>††</sup> F-statistic significant at 75 percent.

June	July	Aug.	Sep.	Oct.	Nov.	Dec.	Trig.
-7.1056***	-5.4819*	-8.4252**		6.8608*			0.2390*
			12.4281*				0.1166*
7.4638**	7.3160*	11.7341***	7.7540*				
					-17.6691*		-0.1527**
							-0.0848*
							0.2388*
56.1796***	36.0051*	63.1635**	48.6044**				0.2689*
	-9.7843*						
				-89.0294*		-62.8913*	
-2.7136**	-2.4601*	-4.3882**	-2.8847*				-0.0184*
0.6563*							
13.1283*						-9.7679*	
	-1.8553*	-2.0918*					-0.0219***
	-39.4205*						
33.08	23.32	28.41	25.08	30.90	22.95	25.65	36.19
0.83	0.51	0.67	0.56	0.75	0.50	0.58	0.95 <sup>+</sup>

Table 10. Final coefficients for the corn basis model, including storage utilization

Variable <sup>a</sup>	Storage utilization	Intercept	Trend	Jan.	Feb.	Mar.	Apr.
Intercept			-0.5270*				
Turnover			-0.1140*				
Storage	-14.5845****	5.5274**		1.7693***			
Employees	-1.9596***	1.3370***					
Concrete	-17.8075*	11.3384*				-4.3027***	-3.3234**
FAPV	17.9528****	-5.6134*		-2.9850****	-2.7542***	-2.3525***	-1.4960*
Distance						-1.3005*	-2.7002****
C gulf							-2.8150*
Pigs							2.4840*
Concentration	-13.5862*						
All cattle			0.1218***	-3.9488*			
Grain cattle			-0.1953****				
Hens							
F concen							
Bushel exp.							
Dry 24							-3.3781**
Shrink							20.9799**
K	-4.8997**			1.0234***	0.9865**	0.8114**	
s	-11.7660**	5.9486*		2.0985***			-1.4178*
t				-0.2189**	-0.2074*	-0.2864***	-0.1991*
TD	21.9093**			-3.8510**			
Cdiv							
LEV							
R <sup>2</sup>	47.10	38.11	45.46	40.14	34.61	41.49	51.04
F-statistic	1.50 <sup>†††</sup>	1.04 <sup>†</sup>	1.40 <sup>††</sup>	1.13 <sup>†</sup>	0.89	1.19 <sup>†</sup>	1.75 <sup>††††</sup>

<sup>a</sup>The variables across the top are time series variables and the variables down the side are cross-sectional variables.

\*t-statistic significant at 80 percent.

\*\*t-statistic significant at 90 percent.

\*\*\*t-statistic significant at 95 percent.

\*\*\*\*t-statistic significant at 99 percent.

<sup>†</sup>F-statistic significant at 50 percent.

<sup>††</sup>F-statistic significant at 75 percent.

<sup>†††</sup>F-statistic significant at 90 percent.

<sup>††††</sup>F-statistic significant at 95 percent.

May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.	Trig.
-0.2207*	-2.0753***	-3.2863***	-2.9162**	-1.9925*	2.2409*	2.2302**		
-2.8594*	-0.4354***	-0.5365***	-0.3603*		0.5232***	0.5089***	0.3341**	
	-3.5726*				5.4972**	4.6682*		0.0656***
	2.5214***	4.4196***	5.1802***	3.9384***	-2.3470*		-2.3240*	0.0194*
	-1.3658*						2.1579**	0.0197**
2.0740*					-4.7584**			
	-2.1382*							
		7.9122**						
		-6.4967*						
			-8.9601*				6.6053*	
	14.5113***							
-4.2419***				6.8490**				
20.2792*								
	-0.8565**	-1.2174**	-1.5674***	-1.2301*				-0.0073*
-1.4492*	-3.5726***	-3.8508***	-3.4213**		3.7510***	2.9100**	4.3141***	0.0158*
		0.4993***	0.4259**	0.5690***			-0.4297***	0.0023*
	4.7233***	8.1119***	5.4890*			-4.8613*	-6.4245***	
45.16	52.68	51.20	39.65	44.09	47.91	38.13	45.12	45.98
1.39 <sup>††</sup>	1.87 <sup>††††</sup>	1.76 <sup>††††</sup>	1.10 <sup>†</sup>	1.33 <sup>††</sup>	1.55 <sup>†††</sup>	1.04 <sup>†</sup>	1.38 <sup>††</sup>	1.43 <sup>††</sup>

Table 11. Final coefficients for the corn basis model, excluding storage utilization

Variables <sup>a</sup>	Intercept	Trend	Jan.	Feb.	March	April	May	June
Intercept		-0.4085*					-28.5361*	
Turnover	-12.5652*		7.1391***			9.8180*	8.7913*	
Storage						2.0469**	1.9011**	
Employees								
Concrete			1.8731*					
FAPV			-1.1213*	-1.6311*	-2.1741*	-2.7720**	-2.4898**	
Distance				-1.3899*		-2.9169***	-1.7401*	
C gulf						-5.2090**	-4.3951**	
Pigs								
Concentration		0.0452**		2.4158*	3.1642*	3.9242***	2.3600*	
All cattle		0.1000***						
Grain cattle	8.3237*	-0.1699****						
Hens						6.0116*		
F concen								
Bushel exp.	4.7066*	-0.0538*						
Dry 24	-6.0972**		2.6150**					-14.4232*
Shrink								
K			0.4510*	0.6920*	0.8410*	0.9254*	0.7542*	
s								
T								
TD								
Cdiv i								
LEV								
R <sup>2</sup>	23.88	31.72	35.53	17.86	17.26	29.52	25.70	13.93
F-statistic	0.63	0.93 <sup>†</sup>	1.10 <sup>†</sup>	0.43	0.42	0.84	0.69	0.32

<sup>a</sup>The variables across the top are time series variables and the variables down the side are cross-sectional variables.

\* t-statistic significant at 80 percent.

\*\* t-statistic significant at 90 percent.

\*\*\* t-statistic significant at 95 percent.

\*\*\*\* t-statistic significant at 99 percent.

<sup>†</sup> F-statistic significant at 50 percent.

<sup>††</sup> F-statistic significant at 75 percent.

<sup>†††</sup> F-statistic significant at 95 percent.

July	Aug.	Sep.	Oct.	Nov.	Dec.	Trig.
-0.1153*			-1.7976**			-0.0253*
1.4009*						
	1.8393**	2.6001**	2.9393***	1.8766*		0.0408**
		2.9253***	3.4704****	1.8224**		0.0356**
-1.4399*			3.9521*			
		-3.7136**	-2.7525*			-0.0418*
6.3858****	4.4506**					
-5.2245****						
			-5.6410*	-5.4179*		
-3.7484**			4.0597*			
					3.3992*	
	-0.7894***	-1.1541**	-1.1169**	-0.8277**		-0.0155**
-1.1896***		2.7577***	2.4470**			0.0251*
0.1307**		0.3698***			-0.2923**	
1.6166*					-4.6192***	
					-0.7782***	
					-11.2923*	
58.64	32.43	37.70	38.99	24.90	23.54	28.22
2.84 <sup>††††</sup>	0.96 <sup>†</sup>	1.21 <sup>††</sup>	1.28 <sup>††</sup>	0.66	0.62	0.79

was 50 percent. Given this level of significance, we can specify the estimated equations for the corn and soybean basis models. To construct the models as specified in Equations 4.3 and 4.4, each column in the table must be analyzed. The intercept of the general model,  $A_0$ , and the cross-sectional coefficients are determined from the intercept column of the time series variables which is derived from the regression of Equation 4.26, i.e.;

$$\hat{D}_{io} = \hat{A}_0 + \sum_{\ell=1}^{23} \hat{\gamma}_{\ell} Z_{i\ell} + \hat{e}_{io} \quad (5.6)$$

where  $\hat{D}_{io}$  is the estimated time series intercept for the i-th firm,  $\hat{A}_0$  is the estimated intercept of the model,  $\hat{\gamma}_{\ell}$  is the estimated coefficient for the cross-sectional variable  $Z_{i\ell}$ , and  $\hat{e}_{io}$  is the residual error for the i-th firm. The  $\hat{\gamma}_{\ell}$  are estimates of the  $\gamma_{\ell}$  in the model specified in Equation 4.3 and  $\hat{A}_0$  is the estimate of  $A_0$  in Equation 4.3.

The  $A_k$  and  $(AY)_{k\ell}$  are estimated by the coefficients derived from regressing the cross-sectional variables on the k-th time series coefficients derived in the time series analyses. The regression of the k-th time series coefficient can be shown as;

$$\hat{D}_{ik} = \hat{A}_k + \sum_{\ell=1}^{23} (\hat{AY})_{k\ell} Z_{i\ell} + \hat{e}_{ik} \quad (5.7)$$

where  $\hat{D}_{ik}$  is the estimate of the k-th time series coefficient for the i-th firm,  $\hat{A}_k$  and  $(\hat{AY})_{k\ell}$  are estimates of  $A_k$  and  $(AY)_{k\ell}$  in Equation 4.3, and  $\hat{e}_{ik}$  is the residual error for the i-th firm in the k-th regression. If the F-statistic is not significant for regression Equation 5.7, then  $\hat{A}_k$  equals the mean value of  $\tilde{D}_{ik}$  (Tables 4 and 5) derived from the time

series regressions, and the  $(\hat{A}\gamma)_k$  equal zero.

For the soybean basis model which includes the storage utilization variable (Table 8) the only time series coefficient which yielded a significant F-statistic in the regression on the cross-sectional elements was the trig coefficient. Interpreting the results as indicated in Equations 5.6 and 5.7 would yield the mean values of the time series coefficients (located in Table 4) as the estimated coefficients for the  $A_0$  and  $A_k$ , except that the coefficient for the trig term would be dependent on the cross-sectional elements as shown in Table 8. Estimating Equation 4.3 in this manner yields the following equation;

$$\begin{aligned} \hat{BB}_{it} = & -27.9045 - 8.2855*SU_{it} - 0.0701*t + 4.9348*S_{1it} \\ & + 5.5948*S_{2it} - 0.2916*S_{3it} + 0.6505*S_{4it} + 2.2133*S_{5it} \\ & + 0.0444*S_{6it} - 2.5492*S_{7it} - 2.0250*S_{8it} - 9.6917*S_{9it} \\ & - 5.5130*S_{10it} + 4.7758*S_{11it} + 1.8568*S_{12it} \\ & + [0.2390*(Turnover)_i + 0.1166*(Concrete) - 0.1527*(S gulf)_i \\ & - 0.0848*(Pigs)_i + 0.2388*(All cattle)_i + 0.2689*(F concen)_i \\ & - 0.0184*(\bar{K})_i - 0.0219*(Sdiv)_i]*(Trig)_{it}. \end{aligned} \quad (5.8)$$

where  $\hat{BB}_{it}$  would be the estimated soybean basis level for a cooperative with cross-sectional variables equal to the i-th firm's cross-sectional variables,  $SU_{it}$  is the level of storage utilization for the i-th firm in the t-th time period.  $S_{kit}$  ( $k=1, \dots, 12$ ) is k-th monthly seasonal dummy



for the  $i$ -th firm in time period  $t$ .  $S_{kit}$  equals 1.0 if the  $t$ -th time period is in month  $k$  and zero otherwise. The variables inside the brackets of the equation are the values of the cross-sectional variables as defined earlier for the  $i$ -th firm.  $(\text{Trig})_{it}$  is the value for the trig time series variable for the  $i$ -th firm in the  $t$ -th time period.

The soybean basis model which excludes the storage utilization variable (Table 9) can be constructed in a manner similar to the soybean basis model including storage utilization. In the soybean basis model, excluding storage utilization, the regression models for the time series coefficients February, April, and August were found to have significant F-statistics at the 50 percent level of significance. Using these regressions for determining the coefficients corresponding to these variables, we can construct a model from Tables 5 and 9 which would contain the average coefficients for the intercept, trend, January, March, May, June, September, October, November, December, and trig terms (found in Table 5); and the regressions found in Table 9 for time series variables February, April, July, and August. An equation may be written for this model as;

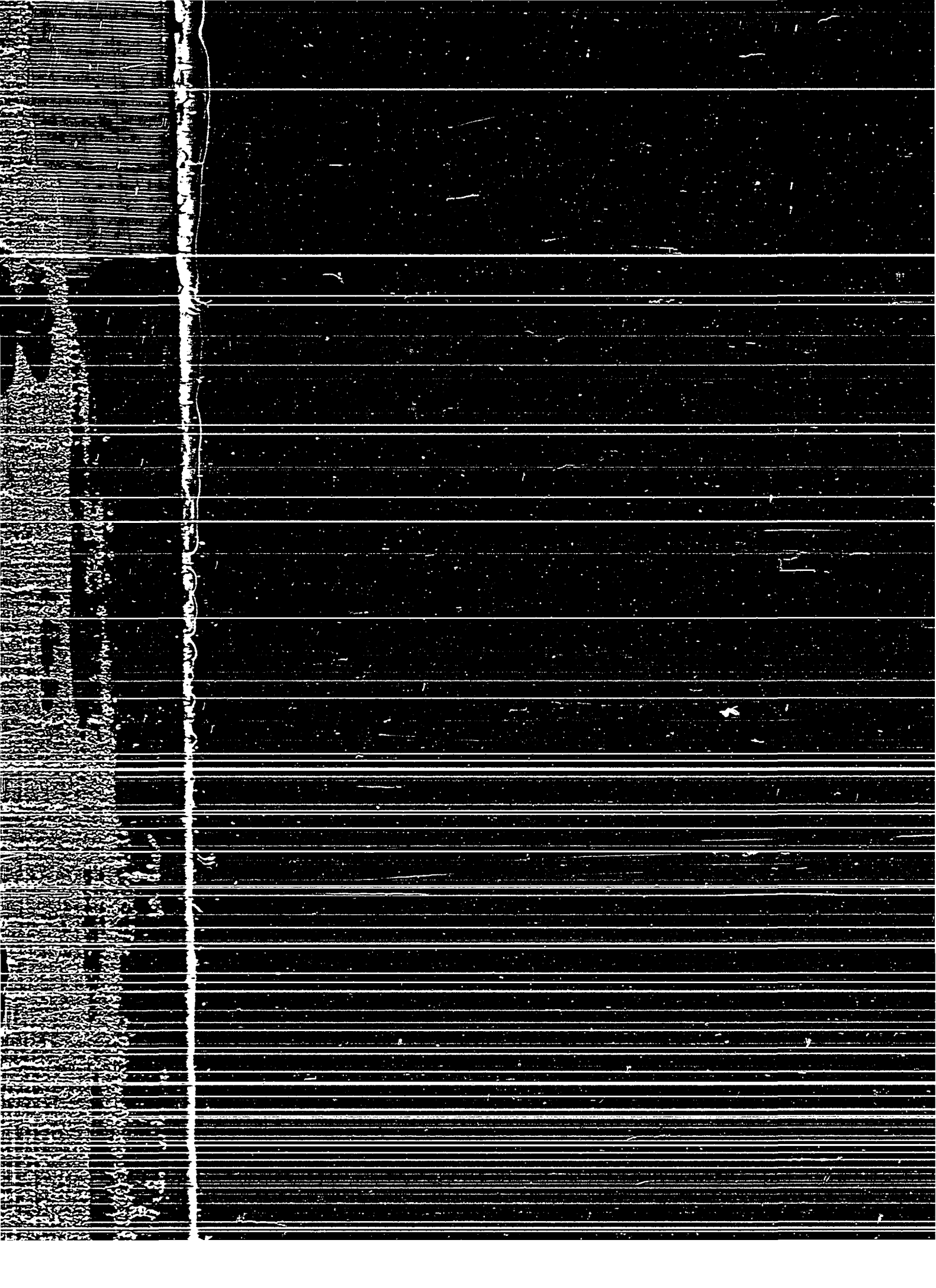
$$\begin{aligned}
\hat{BB}_{it} = & -24.9249 - 0.2393*t - 7.1321*S_{1it} + [-8.8731*(Concrete)_i \\
& - 11.2815*(F \text{ concen})_i - 4.7778*(Dry \ 24)_i]*S_{2it} - 21.7780*S_{3it} \\
& - [74.1645 + 8.8452*(S \ gulf)_i + 7.0601*(Pigs)_i \\
& - 7.2960*(Concentration)_i \\
& - 35.2849*(All \ cattle)_i + 24.9248*(Grain \ cattle)_i \\
& - 3.5411*(s)_i]*S_{4it} - 5.9555*S_{5it} + 1.9903*S_{6it} \\
& + [53.5778 - 4.9231*(Pigs)_i - 5.0545*(Bushel \ expense)_i \\
& - 29.5655*(Shrink)_i + 1.8953*(s)_i]*S_{7it} \\
& + [-0.3803*(Employees)_i + 21.3142*(All \ cattle)_i \\
& - 18.5713*(Grain \ cattle)_i + 4.6033*(Dry \ 24)_i \\
& - 0.9023*(\bar{K})_i]*S_{8it} + 11.1424*S_{9it} + 1.9490*S_{10it} \\
& + 0.4465*S_{11it} + 5.2762*S_{12it} + 0.0949*(Trig)_{it}. \quad (5.9)
\end{aligned}$$

The time series variables in this equation can be interpreted similarly to those in Equation 5.8. The cross-sectional variables located inside each of the brackets in Equation 5.9 are defined for each firm (i) as equal to the definitions given before.

An equation for the corn basis model including the storage utilization variable (Tables 6 and 10) can be constructed in a manner similar to both soybean basis models. In the corn basis model which includes storage utilization the only time series coefficient which did not yield a significant F-statistic was the February coefficient. To construct this corn basis model we would use the average value for the February time

series coefficient (from Table 6) and the regressions as defined for the other time series variables in Table 10, i.e.;

$$\begin{aligned}
 \hat{BC}_{it} = & 5.5274*(Storage)_i + 1.3370*(Employees)_i + 11.3384*(Concrete)_i \\
 & -5.6134*(FAPV)_i \\
 & +5.9486*(s)_i + [-14.5845*(Storage)_i \\
 & -1.9596*(Employees)_i - 17.8075*(Concrete)_i + 17.9528*(FAPV)_i \\
 & -13.5862*(Concentration)_i - 4.8997*(\bar{K})_i - 11.7661*(s)_i \\
 & +21.9093*(TD)_i]*SU_{it} + [-0.5270 + 0.1141*(Turnover)_i \\
 & +0.1218*(All\ cattle)_i - 0.1953*(Grain\ cattle)_i]*t \\
 & + [1.7693*(Storage)_i - 2.9850*(FAPV)_i - 3.9480*(All\ cattle)_i \\
 & +1.0234*(\bar{K})_i + 2.0985*(s)_i - 0.2189*(\tau)_i - 3.8510*(TD)_i]*S_{lit} \\
 & -11.4915*S_{2it} + [-4.3027*(Concrete)_i - 2.3525*(FAPV)_i \\
 & -1.3005*(Distance)_i + 0.8114*(\bar{K})_i - 0.2864*(\tau)_i]*S_{3it} \\
 & +[-3.3235*(Concrete)_i - 1.4960*(FAPV)_i - 2.7002*(Distance)_i \\
 & -2.8150*(C\ gulf)_i + 2.4840*(Pigs)_i - 3.3781*(Dry24)_i \\
 & +20.9709*(Shrink)_i - 1.4178*(s)_i - 0.1991*(\tau)_i]*S_{4it} \\
 & +[-0.2207*(Employees)_i - 2.8594*(Concrete)_i + 2.0740*(Pigs)_i \\
 & -4.2419*(Dry\ 24)_i + 20.2792*(Shrink)_i - 1.4492*(s)_i]*S_{5it} \\
 & +[-2.0753*(Storage)_i - 0.4354*(Employees)_i - 3.5726*(Concrete)_i \\
 & +2.5214*(FAPV)_i - 1.3659*(Distance)_i - 2.1382*(Concentration)_i
 \end{aligned}$$



$$\begin{aligned}
& +14.5113*(F \text{ concen})_i - 0.8565*(\bar{K})_i - 3.5726*(s)_i + 4.7234*(TD)_i] * S_{6it} \\
& +[-3.2863*(Storage)_i - 0.5365*(Employees)_i + 4.4196*(FAPV)_i \\
& +7.9122*(All \text{ cattle})_i - 6.4967*(Grain \text{ cattle})_i - 1.2174*(\bar{K})_i \\
& -3.8508*(s)_i + 0.4993*(\tau)_i + 8.1119*(TD)_i] * S_{7it} \\
& +[-2.9162*(Storage)_i - 0.3603*(Employees)_i + 5.1802*(FAPV)_i \\
& -8.9601*(Hens)_i - 1.5674*(\bar{K})_i - 3.4213*(s)_i + 0.4259*(\tau)_i \\
& +5.4890*(TD)_i] * S_{8it} + [-1.9925*(Storage)_i + 3.9384*(FAPV)_i \\
& +6.8490*(Dry \text{ 24})_i - 1.2301*(\bar{K})_i + 0.5690*(\tau)_i] * S_{9it} \\
& +[2.2409*(Storage)_i + 0.5232*(Employees)_i + 5.4972*(Concrete)_i \\
& -2.3470*(FAPV)_i - 4.7584*(Pigs)_i + 3.7510*(s)_i] * S_{10it} \\
& +[2.2302*(Storage)_i + 0.5089*(Employees)_i + 4.6682*(Concrete)_i \\
& +2.9108*(s)_i - 4.8613*(TD)_i] * S_{11it} + [0.3341*(Employees)_i \\
& -2.3240*(FAPV)_i + 2.1579*(Distance)_i + 6.6053*(Hens)_i \\
& +4.3141*(s)_i - 0.4297*(\tau)_i - 6.4245*(TD)_i] * S_{12it} \\
& + [0.0656*(Concrete)_i + 0.0194*(FAPV)_i + 0.0197*(Distance)_i \\
& -0.0074*(\bar{K})_i + 0.0158*(s)_i + 0.0023*(\tau)_i] * (Trig)_{it}. \tag{5.10}
\end{aligned}$$

In Equation 5.10  $\hat{BC}_{it}$  is the estimated corn basis level for a cooperative with cross-sectional variables equal to the  $i$ -th firm's cross-sectional variables, and the other variables are as defined before.

The final model which was estimated was the corn basis model excluding storage utilization. The F-statistics in Table 11 indicate that the time series coefficient regressions for trend, January, July, August, September, and October were significant. The average value of the time series coefficients (Table 7) would be used for the intercept, February, March, April, May, June, November, December, and Trig. The corn basis equation may be written as;

$$\begin{aligned}
 \hat{BC}_{it} = & -30.3467 + [-0.4085 + 0.0452*(\text{Concentration})_i \\
 & + 0.1001*(\text{All cattle})_i - 0.1699*(\text{Grain cattle})_i \\
 & - 0.0538*(\text{Bushel expense})_i]*t \\
 & + [7.1391*(\text{Turnover})_i + 1.8731*(\text{Concrete})_i - 1.1213*(\text{FAPV})_i \\
 & + 2.6160*(\text{Dry } 24)_i + 0.4511*(\bar{K})_i]*S_{1it} + 4.4816*S_{2it} \\
 & - 1.0691*S_{3it} + 0.4983*S_{4it} + 2.5230*S_{5it} + 3.2811*S_{6it} \\
 & + [-0.1154*(\text{Employees})_i + 1.4010*(\text{Concrete})_i \\
 & - 1.4399*(\text{C gulf})_i + 6.3858*(\text{All cattle})_i \\
 & - 5.2245*(\text{Grain cattle})_i - 3.7484*(\text{F concen})_i - 1.1897*(s)_i \\
 & + 0.1307*(\tau)_i + 1.6166*(\text{TD})_i]*S_{7it} \\
 & + [1.8393*(\text{FAPV})_i + 4.4506*(\text{All cattle})_i - 0.7894*(\bar{K})_i]*S_{8it} \\
 & + [2.6001*(\text{FAPV})_i + 2.9255*(\text{Distance})_i - 3.7136*(\text{Concentration})_i \\
 & - 1.1541*(\bar{K})_i + 2.7577*(s)_i + 0.3698*(\tau)_i]*S_{9it} \\
 & + [-1.7976*(\text{Storage})_i + 2.9392*(\text{FAPV})_i + 3.4704*(\text{Distance})_i \\
 & + 3.9520*(\text{C gulf})_i - 2.7525*(\text{Concentration})_i \\
 & - 5.6410*(\text{Hens})_i + 4.0597*(\text{Bushel expenses})_i \\
 & - 1.1169*(\bar{K})_i + 2.4480*(s)_i]*S_{10it} + 3.2109*S_{11it} \\
 & + 0.1810*S_{12it} - 0.0273*(\text{Trig})_{it}.
 \end{aligned} \tag{5.11}$$

The variables in Equation 5.11 may be interpreted in the same way as in Equations 5.8, 5.9, and 5.10.

No definite conclusions can be drawn from these results about the effect of any of the cross-sectional variables on the basis level of corn or soybeans. The signs of the coefficients on no cross-sectional variable was consistent in all four models. Table 12 shows the summary of the coefficient's signs of the cross-sectional variables.

Table 12. The summary of the coefficients signs of the cross-sectional variables in all basis models

Variable	Soybean basis Equation 5.8	Soybean basis Equation 5.9	Corn basis Equation 5.10	Corn basis Equation 5.11
Turnover	1(+)		1(-)	1(+)
Storage			4(+)	5(-)
Employees		1(-)	4(+)	5(-)
Concrete	1(+)	1(-)	4(+)	5(-)
FAPV			6(+)	6(-)
Distance			2(+)	3(-)
Gulf <sup>a</sup>	1(-)	1(+)		1(-)
Pigs	1(-)	1(+)	1(-)	2(+)
Concentration		1(-)		2(-)
All cattle	1(+)	1(+)	1(-)	2(+)
Grain cattle		1(+)	1(-)	2(-)

<sup>a</sup>Gulf is S gulf in Equations 5.8 and 5.9 and C gulf in Equations 5.10 and 5.11.

Table 12 (Continued)

Variable	Soybean basis Equation 5.8	Soybean basis Equation 5.9	Corn basis Equation 5.10	Corn basis Equation 5.11
Hens			1 (+) 1 (-)	1 (-)
F concn	1 (+)	1 (-)	1 (+)	1 (-)
Bushel expense		1 (-)		1 (+) 1 (-)
Dry 24		1 (+) 1 (-)	1 (+) 2 (-)	1 (+)
Shrink		1 (-)	2 (+)	
$\bar{K}$	1 (-)	1 (-)	2 (+) 6 (-)	1 (+) 3 (-)
s		1 (+) 1 (-)	5 (+) 6 (-)	2 (+) 1 (-)
$\tau$			4 (+) 4 (-)	2 (+)
TD			4 (+) 3 (-)	1 (+)
Div <sup>b</sup>	1 (-)			
LEV				

<sup>b</sup>Div is Sdiv in Equations 5.8 and 5.9 and Cdiv in Equation 5.10 and 5.11.



## CHAPTER VI. COOPERATIVE FINANCING

## SUB-MODEL

The sub-model developed for the pricing and production decision in Chapter III utilized the objective of maximizing the total collective profits of the member patrons. The approach used in most of the literature concerning the cooperative financing decision has been to minimize the cost of capital. Much of the corporate literature has used the objective of maximizing the rate of return on owners' investment in the firm.

The sub-model used in this chapter to analyze cooperative financing will utilize the objective of maximizing the total collective profits of all present members. We can distinguish the financing sub-model from the pricing and production sub-model in the process of decision-making the cooperative decision-maker operates by. The production and pricing sub-model is considered to be a short-run sub-model of the firm for functioning within the limits and in accordance with the requisites of the established structure. The established structure of the cooperative association will be that which is determined in the long-run decision-making process. The enterprise structure analysis will contain two basic decisions. First, the cooperative association must determine the financial structure within which the cooperative must operate. Second, the cooperative association must determine the types and amounts of asset investments to make. The model developed in this chapter will be concerned with the financial structure of the cooperative association.

Determining the types and amounts of asset investments to make will be a separate decision process not considered.

The temporal relationship of the finance sub-model with the production and pricing sub-model assumes that the finance decision is made prior to the production and pricing decision. In the finance sub-model I am assuming that the cooperative association is in a long-run decision process. The long-run is distinguished from the short-run by the variables which the cooperative association can vary. With the production and pricing sub-model developed in Chapter III, we were able to obtain the structural equations of 4.1 and 4.2 as results which gave the price of unprocessed products (set X) and the price of variable inputs (set Y) as dependent upon structural variables within the firm. In the long-run finance decision we will allow these structural variables to be decision variables.

The model of the cooperative association developed in Chapter II (Figure 2.1) is applicable for development of the financing sub-model. The cooperative association is composed of firms joined together for the purpose of processing and marketing their products or supplying the inputs they use in production. The activities may or may not be limited to members. The various relationships of products with member and non-member patrons will still exist.

In this chapter, we will need to reassess the member and nonmember patron's decision process in the long-run. The decision which the patrons will need to make is whether to belong to the cooperative association. To belong to the cooperative association, the member must pay for a share

of common stock and allow the cooperative association the right to retain allocated earnings of the member patron as an additional method of investment in the cooperative. In the long-run membership will not be fixed for the cooperative decision-maker, but dependent upon the operating decisions of the cooperative.

#### Model of a Typical Patron for Determining Membership

The patron of a cooperative association must determine whether to join the cooperative association. In Chapter III, we described the typical patron as a maximizer of profits. A typical decision of a profit maximizer is determining which asset investments, or disinvestments, to make. Authors have used different criteria for determining optimal asset investment, e.g., net present value, rate of return, and profitability index. Most asset investment criteria involve the cash flows to and from the firm. The decision of joining the cooperative association can be assumed to be a rate of return analysis.

The total cash inflow to a cooperative association member patron for the present year may be described as the total sales revenues received or paid for products traded with and outside the cooperative association plus the net present value of patronage dividends allocated to the member patrons plus the dividends on stock, i.e.;

$$\begin{aligned} \Pi_M = & \sum_{i \in X} p_i q_{ic}^M + \sum_{i \in X} p_i q_{io}^{OM} - \sum_{i \in Y} p_i q_{ic}^M - \sum_{i \in Y} p_i q_{io}^{OM} \\ & + [s + \frac{(1-s)}{(1+d_M)}] \tau [ \sum_{i \in X, Y} r_i^* q_{ic}^M ] + ds \end{aligned} \quad (6.1)$$

where  $p_i$  and  $q_{ic}^M$  are the price and quantity of the  $i$ -th product traded with the cooperative association by the member.  $p_i^O$  and  $q_{io}^M$  are the price and quantity of the  $i$ -th product traded outside the cooperative association by the member,  $s$  is the proportion of patronage dividends paid in cash,  $(1-s)$  is the proportion of patronage dividends held for deferred payment in  $\tau$  years,  $d_M$  is the discount rate of the member,  $r_i^*$  is the expected per unit patronage dividend of the  $i$ -th product, and  $ds$  is the dividends on stock.

The cash inflow of the typical nonmember patron for the present year may be described as the total sales revenues received or paid for products traded with and outside the cooperative association, i.e.;

$$\Pi_N = \sum_{i \in X} p_i q_{ic}^N + \sum_{i \in X} p_i^O q_{io}^N - \sum_{i \in Y} p_i q_{ic}^N - \sum_{i \in Y} p_i^O q_{io}^N \quad (6.2)$$

where  $p_i$  and  $q_{ic}^N$  are the price and quantity of the  $i$ -th product traded with the cooperative association by the nonmember and  $p_i^O$  and  $q_{io}^N$  are the price and quantity of the  $i$ -th product traded outside the cooperative association by the nonmember. We may assume the price of the  $i$ -th product ( $p_i$  and  $p_i^O$ ) to be the same in both models. The quantity supplied or demanded by the typical nonmember patron ( $q_{ic}^N$ ) will differ from that of the member patron ( $q_{ic}^M$ ) because of the expectation of patronage dividends by members, and the quantity is assumed to be larger for the member patron.

The net cash inflow that results from becoming a member for the present year is the difference between the cash inflows of the typical member and nonmember patrons, i.e.;

$$\pi_D = \pi_M - \pi_N \quad (6.3)$$

or;

$$\begin{aligned} \pi_D = & \sum_{i \in X} [p_i (q_{ic}^M - q_{ic}^N) + p_i^o (q_{io}^M - q_{io}^N)] - \sum_{i \in Y} [p_i (q_{ic}^M - q_{ic}^N) + p_i^o (q_{io}^M - q_{io}^N)] \\ & + [s + \frac{(1-s)}{(1+d_m)^T}] [\sum_{i \in X, Y} r_i^* q_{ic}^M] + ds. \end{aligned} \quad (6.4)$$

The total member investment required to become a fully invested member in the cooperative association is the sum of the cost of a share of common stock and the total amount of patronage dividends retained. Assuming the member expects a constant amount of patronage dividends each year, we can write this total cash outflow as;

$$MI = P_m + \tau(1-s) \sum_{i \in X, Y} p_i^* q_{ic}^M \quad (6.5)$$

where  $P_m$  is the price of a share of common stock and all other variables are as described before.

Given Equations 6.4 and 6.5 we can describe the rate of return for membership in the cooperative as;

$$RR_M = \frac{\pi_D}{MI} \quad (6.6)$$

or;

$$\begin{aligned} & \sum_{i \in X} [p_i (q_{ic}^M - q_{ic}^N) + p_i^o (q_{io}^M - q_{io}^N)] - \sum_{i \in Y} [p_i (q_{ic}^M - q_{ic}^N) + p_i^o (q_{io}^M - q_{io}^N)] \\ & + [s + \frac{(1-s)}{(1+d_m)^T}] [\sum_{i \in X, Y} r_i^* q_{ic}^M] + ds. \\ RR_M = & \frac{\sum_{i \in X} [p_i (q_{ic}^M - q_{ic}^N) + p_i^o (q_{io}^M - q_{io}^N)] - \sum_{i \in Y} [p_i (q_{ic}^M - q_{ic}^N) + p_i^o (q_{io}^M - q_{io}^N)] + [s + \frac{(1-s)}{(1+d_m)^T}] [\sum_{i \in X, Y} r_i^* q_{ic}^M] + ds.}{P_m + \tau(1-s) \sum_{i \in X, Y} r_i^* q_{ic}^M} \end{aligned} \quad (6.7)$$

If the patron were to use the rate of return method for determining membership, he would compare  $RR_M$  with  $d_m$ . If  $RR_M$  were greater than  $d_m$ , then the patron would become a cooperative member.

The other methods of determining membership (net present value and profitability index) would utilize the same variables developed above. In considering only the one investment option, all methods should give the same result. If we were considering a group of investments, the ranking of investments could differ, depending on the method of analysis. The only investment option I will consider will be membership in the cooperative association. Given this assumption, each patron could determine his decision for membership with Equation 6.7 and his discount rate ( $d_M$ ). Total membership would be the aggregate of the decisions made by all patrons. All patrons will not make the same decision since each faces a different set of exogenous variables.

The  $p_i^O$ ,  $q_{ic}^M$ ,  $q_{ic}^N$ ,  $d_M$ , and  $r_i^*$  could and probably would differ for each patron, and therefore dictate different decisions. Membership in the cooperative will therefore depend upon the value of these variables for the patrons it serves, i.e., total membership may be expressed by;

$$M = M(p_X, p_X^O, p_Y, p_Y^O, Q_G, s, \tau, d_M, R_G, ds, p_m) \quad (6.8)$$

where  $p_X$  is the vector of prices paid by the cooperative association for the products in set X,  $p_X^O$  is the vector of prices paid by firms outside the cooperative association for products in set X,  $p_Y$  is the vector of prices received by the cooperative association for products in set Y,  $p_Y^O$  is the vector of prices received by firms outside the cooperative association for products in set Y,  $Q_G$  is the vector of quantities of public goods produced by the cooperative association,  $R_G$  is the vector of expected per unit patronage dividend rates of the patrons, and  $s, \tau, d_M, p_m$

and  $ds$  are as described before. The  $q_{ic}^M$  and  $q_{ic}^N$  were eliminated from the equation because of their dependence on  $P_X, P_Y, Q_G$ , and  $R_C$ .

#### Model of the Cooperative Association

In the production and pricing sub-model of the cooperative association, we assumed that the cooperative decision-maker had the objective of maximizing the collective profits of the member patrons, i.e., the objective function was;

$$\Pi = \sum_{i \in X} p_i q_i^{TM} - \sum_{i \in Y} p_i q_i^{TM} - FCM + DS + PVPD. \quad (3.26)$$

The long-run objective of the cooperative decision-maker will also be to maximize the total collective profits of the member patrons. In the long-run we will have different variables which are decision variables to the cooperative decision-maker. The long-run decision will involve the establishment of the optimal structure for operating the cooperative. Optimal structural analysis includes asset investment analysis and financial structure. The structural decision which is analyzed in this chapter concerns the financial structure decision.

In determining the optimal financial structure of the cooperative, the cooperative decision-maker must determine the total amount of debt, total amount of owners' equity, and the specific amounts of alternative types of owners' equity to employ. Owners' equity in cooperative associations includes common stock and deferred patronage dividends. Membership allows the patron two important rights. First, it allows the members to vote at annual meetings and have the right to be elected to

the board of directors. This allows patrons the right to be involved in the control of the cooperative enterprise. Second, it is required for the patron to receive a patronage dividend at the end of the fiscal year by most cooperatives.

The net savings of the cooperative association are allocated to the members either as dividends on stock or as patronage dividends, i.e.;

$$NS = PD + DS \quad (6.9)$$

where PD is the total amount of net savings allocated as patronage dividends, and DS is the total amount of net savings allocated as dividends on stock. Patronage dividends can be allocated in cash, deferred patronage dividends, or preferred stock. The cash portion of the allocation is determined by the board of directors of the cooperative provided certain provisions have been met. Under the Iowa Code beginning January 1, 1980, the cooperative must have at least 20 percent cash allocation of present patronage dividends. The cooperative may allocate a portion greater than 20 percent, provided there are no deferred patronage dividends of deceased members in the cooperative.<sup>1</sup> The portion of the allocation not paid in cash would go to either deferred patronage dividends or preferred stock. Allocating preferred stock allows the cooperative the choice of cash allocations greater than 20 percent without restrictions in the manner the noncash portion is

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<sup>1</sup>The cooperative must pay a 20 percent cash allocation if there are deferred patronage dividends of deceased members in the cooperative. Iowa Code (13, Section 30).



paid. Deferred patronage dividends must be paid to the members with the oldest deferred patronage dividend being held by the cooperative paid first. The major difference between deferred patronage dividends and preferred stock is the increased freedom allowed in retiring the preferred stock. I will assume that preferred stock will be handled the same as deferred patronage dividends and I will also assume that no deferred patronage dividends of deceased members is held by the cooperative.

Paying dividends on stock is a method of paying a dividend on member investment in the firm. One of the principles of cooperation states that the cooperative should pay only a limited return on investment, i.e., limited to a fixed rate. The dividend could be defined as a fixed dividend rate on total member investment from common stock, i.e.;

$$DS = i[CS] \quad (6.10)$$

where  $i$  is the fixed dividend rate and  $CS$  is the total member investment in common stock.

The total amount of member investment in common stock is dependent upon the number of members ( $M$ ) and the price of common stock ( $P_m$ ). Each member is allowed one share of common stock under the rules of the Internal Revenue Service governing cooperatives, i.e., the number of shares equals the number of members. The number of members was found to be determined by Equation 6.2 in the model of the typical patron. The price of common stock is fixed in the articles and may be changed by

amending the articles of incorporation. Amending the articles of incorporation requires a two-thirds affirmative vote from all members present at the annual meeting or any meeting held for that purpose.<sup>1</sup> The average price of stock can be changed by issuing a different class of preferred stock. Many cooperatives have provisions in the articles of incorporation that allow the board of directors to issue a second class of stock to cause a change in the average price of the stock. This method also gives the advantage of offering a low initial common stock price so that members could pay it outright and receive the benefits of full membership while earning a second class of stock from their patronage dividends received from the cooperative. Given these definitions, we can define the member investment by common stock as;

$$CS = P_m \cdot M \quad (6.11)$$

and redefine the dividend on total common stock investment as,

$$DS = i[P_m M]. \quad (6.12)$$

The other method of obtaining member investment in the cooperative is by deferred patronage dividends. Given a cash allocation of  $s$ , the deferred portion of the patronage dividend is;

$$DP = (1-s)[NS-DS]. \quad (6.13)$$

Once the optimum capital structure is defined, membership will be fixed until a change in one of the exogenous variables occurs. Given the

<sup>1</sup>Iowa Code (13, Section 41).

level of membership, and no expected change in the exogenous variables, we may define the members' investment via the deferred portion of patronage dividends to be;

$$TDP = \tau(1-s)(NS-DS), \quad (6.14)$$

and total member investment as;

$$TMI = \tau(1-s)(NS-DS) + P_m M. \quad (6.15)$$

The total capital employed (K) by the firm is the sum of the total member investment and debt (D), i.e.;

$$K = \tau(1-s)(NS-DS) + P_m M + D. \quad (6.16)$$

The net savings of the cooperative can alternatively be broken down into operating income and interest expense. Operating income (O) is the excess of revenues over costs (not including the total interest cost of debt). I will assume that the operating income is dependent on total membership (M) and total capital employed (K), i.e., net savings may be expressed as;

$$NS = O(M,K) - rD \quad (6.17)$$

where  $r$  is the average interest rate on all debt and  $D$  is the total amount of debt employed. The average interest rate will be dependent upon the leverage ratio of the firm ( $D/K$ ). I will assume the average interest rate is positively related to the leverage ratio. Vickers<sup>1</sup>

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<sup>1</sup>D. Vickers (57, pp. 67-69).

shows how this might be proven by showing how the higher leverage ratio implies a higher amount of risk assumed by the lending institution and, therefore, a higher interest rate.

By using Equations 6.12 and 6.17 we can express the patronage dividends (PD) from Equation 6.9 as;

$$PD = O(M,K) - rD - iP_m M \quad (6.18)$$

and the present value of patronage dividends (PVPD) as;

$$PVPD = [s + \frac{(1-s)}{(1+d_m)^{\tau}}] [O(M,K) - rD - iP_m M]. \quad (6.19)$$

The terms  $s + (1-s)/(1+d_m)^{\tau}$  define the present value of the patronage dividend allocated with portion  $s$  paid in cash and  $(1-s)$  paid  $\tau$  years later.  $d_m$  is the average annual discount rate of the membership. Denoting net savings as expressed in Equation 6.9 and noting that dividend on common stock is paid in cash, then the present value of net savings (PVNS) is the sum of the present value of patronage dividends (Equation 6.19) and dividend on common stock (Equation 6.12), i.e.;

$$PVNS = [s + \frac{(1-s)}{(1+d_m)^{\tau}}] [O(M,K) - rD - iP_m M] + iP_m M. \quad (6.20)$$

The objective of the cooperative is to maximize the total collective profits of its members. Equation 3.26 shows the objective function of the cooperative association for the short-run production and pricing sub-model. In the long-run we allow the structural parameters of the cooperative to become decision variables. The total sales revenues to the member patrons received or paid for products in set  $X$  and set  $Y$  will

vary with the level of membership and total capital employed, i.e.;

$$\sum_{i \in X} p_i q_{ic}^{TM} - \sum_{i \in Y} p_i q_{ic}^{TM} = T(M, K) \quad (6.21)$$

Given Equation 6.21 for total sales revenues to the member patrons, we can redefine the objective function to be;

$$\Pi_M^F = T(M, K) + [s + \frac{(1-s)}{(1+d_m)^T}] [NS-DS] + DS - FCM$$

where NS is defined in Equation 6.17 and DS is defined in Equation 6.12.

Since membership is allowed to vary, it is possible that increasing membership in the cooperative association indefinitely would increase the total collective profits as long as  $T(M, K)$  is positively related to membership ( $M$ ) and not offset by decreases in net savings (NS). Furubotn<sup>1</sup> took this factor into account in his long-run analysis of the labor-managed firm. He assumed the objective of the cooperative was to maximize the total profits of the present members only. Since those members within the cooperative are profit maximizers, it is a fair assumption in this model to assume that they are interested in maximizing their own profits and not concerned with those patrons who might become members. Noting the total number of members to be  $M_0$  now and  $M$  after the optimal capital structure is established, we can specify the long-run objective function to be;

$$\Pi_{M_0}^F = \Pi_M^F \cdot \frac{M_0}{M} \quad (6.23)$$

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<sup>1</sup>See E. G. Furubotn (22, p. 107).

or;

$$\pi_{M_0}^F = \frac{M_0}{M} \left\{ T(M, K) + \left[ s + \frac{(1-s)}{(1+d_m)} \right] [NS-DS] + DS \right\}. \quad (6.24)$$

The long-run objective function of the cooperative association will be subject to five constraints. We can define the total capital employed by the cooperative association (Equation 6.16) to be a definitional constraint. The second constraint specifies that the present membership will remain as members, i.e.,

$$M \geq M_0. \quad (6.25)$$

This constraint is specified since the cooperative decision-maker is concerned with maximizing the total collective profits of the present members. Total membership will be the most appropriate measure to guarantee that present membership has this objective satisfied. This constraint can also be justified by assuming that the present members have already analyzed cooperative association membership as profitable and will retain membership within the firm because they are in essence the policy-makers for the cooperative. Given this assumption, membership with optimum financing will contain all old members and new members who find the cooperative association membership profitable.

The third and fourth constraints specify that the proportion of patronage dividends paid in cash will be greater than or equal to 20 percent but less than or equal to 100 percent, i.e.;

$$s \geq .2 \quad (6.26)$$

$$1.0 \geq s. \quad (6.27)$$

These constraints are derived from the laws governing cooperatives.<sup>1</sup> We will allow  $s$  to be greater than 20 percent since I have assumed the cooperative association holds no deferred patronage dividends of deceased members.

The final constraint will limit the dividend rate allowed on common stock investment. Currently, Iowa law<sup>2</sup> limits the annual dividend rate to no more than eight percent on common stock, i.e.;

$$.08 \geq i. \quad (6.28)$$

From the objective function (6.24) and the constraints (6.16, 6.25, 6.26, 6.27, and 6.28), we can define the Lagrangian for this sub-model to be;

$$\begin{aligned} \Lambda = & \frac{M}{M} \{ T(M, K) + [s + \frac{(1-s)}{(1+d_m)} \tau] [NS-DS] + DS \} \\ & + \phi_1 [K - P_m M - \tau (1-s) (NS-DS) - D] \\ & + \phi_2 (M - M_0) + \phi_3 (s - .2) + \phi_4 [1.0 - s] + \phi_5 [.08 - i] \end{aligned} \quad (6.29)$$

where  $\phi_1$  is the Lagrange multiplier corresponding to the total capital employed,  $\phi_2$  is the Lagrange multiplier corresponding to the membership constraint,  $\phi_3$  and  $\phi_4$  are the Lagrange multipliers corresponding to  $s$ ,  $\phi_5$  is the Lagrange multiplier corresponding to  $i$ ,  $NS$  is defined by Equation 6.17,  $DS$  is defined by Equation 6.12,  $M$  is defined in Equation 6.8, and all

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<sup>1</sup>Iowa Code (13, Section 30).

<sup>2</sup>Iowa Code (13, Section 23).

other variables are as defined before. The decision variables available to the cooperative decision-maker include the total capital employed, the price of a share of common stock, the proportion of patronage dividends paid in cash, the length of the revolving fund, total debt employed, and the dividend rate on total member investment. The Kuhn-Tucker conditions, whose economic interpretations will be presented in the next chapter, are;

$$\begin{aligned} \frac{\partial \Lambda}{\partial K} = & \frac{M_0}{M} \left\{ \frac{\partial T(M, K)}{\partial K} + \left[ s + \frac{(1-s)}{(1+d_m)} \tau \right] \left[ \frac{\partial O}{\partial K} + \frac{D^2}{K^2} \frac{\partial r}{\partial \frac{D}{K}} \right] \right\} \\ & + \phi_1 \left\{ 1 - \tau(1-s) \left[ \frac{\partial O}{\partial K} + \frac{D^2}{K^2} \frac{\partial r}{\partial \frac{D}{K}} \right] \right\} = 0 \end{aligned} \quad (6.30)$$

$$\begin{aligned} \frac{\partial \Lambda}{\partial s} = & - \frac{M_0}{M^2} \frac{\partial M}{\partial s} \Pi_M^F + \frac{M_0}{M} \left\{ \frac{\partial T(M, K)}{\partial M} \frac{\partial M}{\partial s} + \left[ 1 - \frac{1}{(1+d_m)} \tau \right] [NS-DS] \right. \\ & + \left. \left[ s + \frac{(1-s)}{(1+d_m)} \tau \right] \left[ \frac{\partial O}{\partial M} \frac{\partial M}{\partial s} - i P_m \frac{\partial M}{\partial s} \right] + i P_m \frac{\partial M}{\partial s} \right\} \\ & - \phi_1 \left\{ P_m \frac{\partial M}{\partial s} - \tau [NS-DS] + \tau(1-s) \left[ \frac{\partial O}{\partial M} \frac{\partial M}{\partial s} - i P_m \frac{\partial M}{\partial s} \right] \right\} \\ & + \phi_2 \left[ \frac{\partial M}{\partial s} \right] + \phi_3 - \phi_4 \geq 0 \end{aligned} \quad (6.31a)$$

$$\frac{\partial \Lambda}{\partial s} \cdot s = 0 \quad (6.31b)$$

$$s \geq 0 \quad (6.31c)$$



$$\begin{aligned}
\frac{\partial \Lambda}{\partial \tau} = & -\frac{M_O}{M^2} \frac{\partial M}{\partial \tau} \Pi_M^F + \frac{M_O}{M} \left\{ \frac{\partial T(M, K)}{\partial M} \frac{\partial M}{\partial \tau} - \left[ \frac{(1-s)}{(1+d_m)} \tau \cdot \ln(1+d_m) \right] [NS-DS] \right. \\
& + \left[ s + \frac{(1-s)}{(1+d_m)} \tau \right] \left[ \frac{\partial O}{\partial M} \frac{\partial M}{\partial \tau} - i P_m \frac{\partial M}{\partial \tau} \right] + i P_m \frac{\partial M}{\partial \tau} \} \\
& - \phi_1 \left\{ P_m \frac{\partial M}{\partial \tau} + (1-s) [NS-DS] + \tau(1-s) \left[ \frac{\partial O}{\partial M} \frac{\partial M}{\partial \tau} - i P_m \frac{\partial M}{\partial \tau} \right] \right. \\
& \left. + \phi_2 \left[ \frac{\partial M}{\partial \tau} \right] \geq 0 \right. \quad (6.32a)
\end{aligned}$$

$$\frac{\partial \Lambda}{\partial \tau} \cdot \tau = 0 \quad (6.32b)$$

$$\tau \geq 0 \quad (6.32c)$$

$$\begin{aligned}
\frac{\partial \Lambda}{\partial D} = & \frac{M_O}{M} \left\{ \left[ s + \frac{(1-s)}{(1+d_m)} \tau \right] \left[ -r - \frac{D}{K} \frac{\partial r}{\partial D} \right] - \phi_1 \left\{ \tau(1-s) \left[ -r - \frac{D}{K} \frac{\partial r}{\partial D} \right] + 1 \right\} \right. \\
& \left. + 1 \right\} \geq 0 \quad (6.33a)
\end{aligned}$$

$$\frac{\partial \Lambda}{\partial D} \cdot D = 0 \quad (6.33b)$$

$$D \geq 0 \quad (6.33c)$$

$$\begin{aligned}
\frac{\partial \Lambda}{\partial P_m} = & -\frac{M_O}{M^2} \frac{\partial M}{\partial P_m} \Pi_M^F + \frac{M_O}{M} \left\{ \frac{\partial T(M, K)}{\partial M} \frac{\partial M}{\partial P_m} + \left[ s + \frac{(1-s)}{(1+d_m)} \tau \right] \left[ \frac{\partial O}{\partial M} \frac{\partial M}{\partial P_m} \right. \right. \\
& \left. \left. - i M - i P_m \frac{\partial M}{\partial P_m} \right] + i M + i P_m \frac{\partial M}{\partial P_m} \right\} \\
& - \phi_1 \left\{ M + P_m \frac{\partial M}{\partial P_m} + \tau(1-s) \left[ \frac{\partial O}{\partial M} \frac{\partial M}{\partial P_m} - i M - i P_m \frac{\partial M}{\partial P_m} \right] \right. \\
& \left. + \phi_2 \left[ \frac{\partial M}{\partial P_m} \right] \geq 0 \right. \quad (6.34a)
\end{aligned}$$

$$\frac{\partial \Lambda}{\partial P_m} \cdot P_m = 0 \quad (6.34b)$$

$$P_m \geq 0 \quad (6.34c)$$

$$\begin{aligned} \frac{\partial \Lambda}{\partial i} = & -\frac{M_o}{M^2} \frac{\partial M}{\partial i} \Pi_M^F + \frac{M_o}{M} \left\{ \frac{\partial T(M,K)}{\partial M} \frac{\partial M}{\partial i} + \left[ s + \frac{(1-s)}{(1+d_m)} \tau \right] \left[ \frac{\partial O}{\partial M} \frac{\partial M}{\partial i} \right. \right. \\ & \left. \left. - P_m M - i P_m \frac{\partial M}{\partial i} \right] + P_m M + i P_m \frac{\partial M}{\partial i} \right\} \\ & - \phi_1 \left\{ P_m \frac{\partial M}{\partial i} + \tau(1-s) \left[ \frac{\partial O}{\partial M} \frac{\partial M}{\partial i} - P_m M - i P_m \frac{\partial M}{\partial i} \right] \right\} \\ & + \phi_2 \left[ \frac{\partial M}{\partial i} \right] - \phi_5 \geq 0 \end{aligned} \quad (6.35a)$$

$$\frac{\partial \Lambda}{\partial i} \cdot i = 0 \quad (6.35b)$$

$$i \geq 0 \quad (6.35c)$$

$$\frac{\partial \Lambda}{\partial \phi_1} = K - P_m M - \tau(1-S)(NS-DS) - D = 0 \quad (6.36)$$

$$\frac{\partial \Lambda}{\partial \phi_2} = M - M_o \geq 0 \quad (6.37a)$$

$$\frac{\partial \Lambda}{\partial \phi_2} \cdot \phi_2 = 0 \quad (6.37b)$$

$$\phi_2 \geq 0 \quad (6.37c)$$

$$\frac{\partial \Lambda}{\partial \phi_3} = s - .2 \geq 0 \quad (6.38a)$$

$$\frac{\partial \Lambda}{\partial \phi_3} \cdot \phi_3 = 0 \quad (6.38b)$$

$$\phi_3 \geq 0 \quad (6.38c)$$

$$\frac{\partial \Lambda}{\partial \phi_4} = 1-s \geq 0 \quad (6.39a)$$

$$\frac{\partial \Lambda}{\partial \phi_4} \cdot \phi_4 = 0 \quad (6.39b)$$

$$\phi_4 \geq 0 \quad (6.39c)$$

$$\frac{\partial \Lambda}{\partial \phi_5} = .08 - i \geq 0 \quad (6.40a)$$

$$\frac{\partial \Lambda}{\partial \phi_5} \cdot \phi_5 = 0 \quad (6.40b)$$

$$\phi_5 \geq 0. \quad (6.40c)$$

## CHAPTER VII. COOPERATIVE FINANCING

## SUB-MODEL ANALYSIS

Interpreting the Lagrange  
Multipliers

Before analyzing the Kuhn-Tucker conditions, the meaning of the Lagrange multipliers must be determined. In general terms, the value of the Lagrange multiplier of a solution indicates how the value of the objective function will change given a one unit change in the constraint constant.  $\phi_1$  will indicate how the value of the objective function will change given a one unit change in the amount of capital employed, i.e.;

$$\phi_1 = \frac{\partial \Pi_M^F}{\partial K} \begin{matrix} > \\ < \end{matrix} 0 \quad (7.1)$$

$\phi_2$  indicates how the collective profits of the present membership will change if we allow minimum membership to decrease one unit, i.e.;

$$\phi_2 = - \frac{\partial \Pi_M^F}{\partial M_0} \geq 0. \quad (7.2)$$

$\phi_3$  and  $\phi_4$  indicate how the objective function will change given a one unit change in the minimum or maximum value of  $s$ , i.e.;

$$\phi_3 = - \frac{\partial \Pi_M^F}{\partial s_{\min}} \geq 0, \quad (7.3)$$

and,

$$\phi_4 = \frac{\partial \Pi_M^F}{\partial s_{\max}} \geq 0. \quad (7.4)$$

One of these, or both, must be equal to zero. If both  $\phi_3$  and  $\phi_4$  are equal to zero, then the solution for  $s$  is within the range of  $s_{\min}$  to  $s_{\max}$ .  $\phi_5$  indicates how the value of the objective function will change given a one unit increase in the maximum allowed value of  $i$ , i.e.;

$$\phi_5 = \frac{\partial \Pi_{M_0}^F}{\partial i_{\max}} \geq 0. \quad (7.5)$$

#### The General Financing Sub-Model

Whenever there is a set of (a), (b), and (c) Kuhn-Tucker conditions, the (c) condition requires that the instrument variable to which the conditions correspond must be nonnegative. In many cases, it is useful to assume the instrument variable has a positive value, in which case condition (b) guarantees that the (a) condition is satisfied as an equality.

Since capital is defined in the first constraint with an equality, Equation 6.30 is satisfied as an equality and may be rewritten as;

$$\begin{aligned} \frac{\partial \Lambda}{\partial K} = & \frac{M_0}{M} \left\{ \frac{\partial T(M, K)}{\partial K} + \left[ s + \frac{(1-s)}{(1-d_m)} \tau \right] \left[ \frac{\partial O(M, K)}{\partial K} + \frac{D^2}{K^2} \frac{\partial r}{\partial D/K} \right] \right\} \\ & + \phi_1 \left\{ 1 - \tau(1-s) \left[ \frac{\partial O(M, K)}{\partial K} + \frac{D^2}{K^2} \frac{\partial r}{\partial D/K} \right] \right\} = 0. \end{aligned} \quad (7.6)$$

The term  $\partial T(M, K)/\partial K$  is the variation in total sales revenues induced by  $dK$ . This is the derivative of the right hand side of Equation 6.21 with respect to capital ( $K$ ). The terms

$\partial O(M,K)/\partial K + [D^2/K^2][\partial r/\partial (D/K)]$  can be interpreted as the variation in net savings (defined in Equation 6.17) induced by  $dK$  and can be redefined as marginal net savings due to  $dK$ ,  $MNS_K$ . The term  $s + (1-s)/(1+d_m)^\tau$  is the present value factor for patronage dividends. This present value factor with the  $MNS_K$  in the first line of Equation 7.6 can be interpreted as the variation in the present value of net savings to the present membership induced by  $dK$ . This is equivalent to the derivative of PVNS in Equation 6.20 with respect to capital ( $K$ ).

The term  $\tau(1-s)$  is the factor for the amount of capital supplied by deferred patronage dividends. This can be shown by  $(1-s)$  being the portion of patronage dividends deferred each year and  $\tau$  being the total years in which the deferred portion of patronage dividends is held. This factor along with  $MNS_K$  represents the variation in the amount of capital supplied by deferred patronage dividends which is induced by  $dK$ .

From our interpretation of the Lagrange multipliers we found;

$$\phi_1 = \frac{\partial \Pi_M^F}{\partial K} \quad (7.7)$$

or, taking the derivative of  $\Pi_M^F$  with respect to  $K$ ;

$$\phi_1 = \frac{M}{M} \left\{ \frac{\partial T(M,K)}{\partial K} + \left[ s + \frac{(1-s)}{(1+d_m)^\tau} \right] [MNS_K] \right\}. \quad (7.8)$$

Substituting  $\phi_1$  for the terms in the first line of Equation 7.6, we can find that;

$$\frac{\partial \Lambda}{\partial K} = \phi_1 + \phi_1 \{1 - \tau(1-s)MNS_K\} = 0 \quad (7.9)$$

or;

$$\frac{\partial \Lambda}{\partial K} = \phi_1 \{2 - \tau(1-s)MNS_K\} = 0. \quad (7.10)$$

From Equation 7.10 we can determine that if  $\phi_1 \neq 0$ , at the optimum;

$$MNS_K = \frac{2}{\tau(1-s)}. \quad (7.11)$$

This condition is equivalent to stating that, for a maximum, the marginal net savings from total capital employed should be equal to exactly twice the inverse of the factor for the amount of capital supplied by deferred patronage dividends. The reason for the marginal net savings from capital employed being twice the inverse of the factor is not exactly clear. This effect may be caused by the presence of direct and indirect effects of capital employment on the total collective profits of the member patrons. The direct effect is the effect of capital on net savings. The indirect effect comes through the effect on deferred patronage dividends. As net savings change so will the total amount of capital supplied from deferred patronage dividends, which in turn effects the level of total collective profits. Expression 7.11 also implies some important characteristics for the variation in net savings induced by  $dK$ , i.e., as long as  $\tau$  does not equal zero and  $s$  does not equal one,<sup>1</sup> then  $MNS_K$  must always be positive. This implies that the cooperative decision-maker should not maximize net savings derived by capital employment since  $MNS_K$  never equals zero. This reaffirms an important distinguishing characteristic of this model of cooperative associations, that is, the objective of this model is not to maximize the net savings in the operation, but to determine a capital structure which

<sup>1</sup>If  $\tau=0$ , then  $s$  equals 1.0 since deferred patronage dividends are not, in fact, deferred.

will maximize the total collective profits of the membership.

We know from Equation 6.38a, that  $s$  must be greater than or equal to .2, i.e., Equation 6.31a must be satisfied by an equality and may be rewritten as;

$$\begin{aligned} \frac{\partial \Lambda}{\partial s} = & \frac{\partial M}{\partial s} \left[ -\frac{M_O}{M^2} \Pi_M^F + \frac{M_O}{M} \left\{ \frac{\partial T(M, K)}{\partial M} + \left[ s + \frac{(1-s)}{(1+d_m)} \tau \right] \left[ \frac{\partial O(M, K)}{\partial M} - i_{P_m} \right] + i_{P_m} \right\} \right. \\ & - \phi_1 \{ P_m + \tau(1-s) \left[ \frac{\partial O(M, K)}{\partial M} - i_{P_m} \right] + \phi_2 \} \\ & + \frac{M_O}{M} \left\{ \left[ 1 - \frac{1}{(1+d_m)\tau} \right] [NS-DS] \right\} \\ & - \phi_1 [-\tau(NS-DS)] \\ & \left. + \phi_3 - \phi_4 = 0. \right. \end{aligned} \quad (7.12)$$

The terms inside the brackets on the first two lines of Equation 7.12 represent the variation in the total collective profits of the present membership from changing membership, which in Equation 7.12 is induced by  $ds$ . This expression contains the derivative of the total collective profits (Equation 6.22) with respect to membership and could be called  $(\partial TCP / \partial M) (\partial M / \partial s)$ . The effects of changing membership comes from:

- 1) the variation of the distribution of profits over a changing membership,  $-\frac{M_O}{M^2} \Pi_M^F$ , which contains the derivative of the right hand term of Equation 6.23 with respect to  $M$ ;
- 2) the variation of the total private sales revenues,



$\frac{M}{M} \frac{\partial T(M,K)}{\partial M}$ , which contains the derivative of the right hand side of Equation 6.21 with respect to M;

- 3) the variation in the present value of patronage dividends,

$\frac{M}{M} [s + \frac{(1-s)}{(1+d_m)^\tau}] [\frac{\partial O(M,K)}{\partial M} - iP_m]$ , which contains the derivative of Equation 6.19 with respect to M;

- 4) the variation in the dividends on common stock,  $\frac{M}{M} iP_m$ , which contains the derivative of Equation 6.12 with respect to M;

- 5) the variation in total collective profits of the present membership from a change in total capital employed,

$-\phi_1 \{P_m - \tau(1-s) [\frac{\partial O(M,K)}{\partial M} - iP_m]\}$ , which contains the derivative of Equation 6.16 with respect to M; and

- 6) the effect of the membership constraint should membership be constrained to  $M_0$ ,  $\phi_2$ .

The term in the third line of Equation 7.12 can be interpreted as the variation in the present value of the net savings to the present membership which is induced by  $ds$ . This contains the derivative of the present value of net savings (Equation 6.20) with respect to  $s$  ( $M$  held constant).  $[1 - \frac{1}{(1+d_m)^\tau}]$  is the variation in the present value factor induced by  $ds$  and causes the effect here. This term could be called  $\frac{M}{M} \frac{\partial PVNS}{\partial s}$ .

The term in the fourth line of Equation 7.12 is the variation in the total collective profits of the present membership arising from a change in total capital provided by the present membership, which is

induced by  $ds$ . This term could be represented by  $\frac{\partial TCP}{\partial K} \frac{\partial K}{\partial s}$ . The portion of this term inside the brackets [ ] is the derivative of total capital provided by deferred patronage dividends<sup>1</sup> (Equation 6.14) with respect to  $s$ .  $\phi_1$  is the variation in total collective profits of present membership induced by a change in total capital employed.

The terms in the last line of Equation 7.12 are the Lagrange multipliers corresponding to  $s$ . These Lagrange multipliers would have an effect on the total collective profits of the present membership if the solution for  $s$  was on the boundary of  $s_{\min}$  (.2) or  $s_{\max}$  (1.0).  $\phi_3$  and  $\phi_4$  represent the shadow price of  $s_{\min}$  and  $s_{\max}$ . These terms can be represented by  $SP_s$  would be nonnegative.

Given the interpretations of the terms in Equation 7.12, we can rewrite the equation as;

$$\frac{\partial \Lambda}{\partial s} = \frac{\partial TCP}{\partial M} \frac{\partial M}{\partial s} + \frac{M}{M} \frac{\partial PVNS}{\partial s} + \frac{\partial TCP}{\partial K} \frac{\partial K}{\partial s} + SP_s. \quad (7.13)$$

This equation is equivalent to stating that, for a maximum, the sum of: a) the variation in the total collective profits of the present membership from changing membership which is induced by  $ds$ ; b) the variation in the present value of the net saving to the present membership which is induced by  $ds$ ; c) the variation in the total collective profits of the present membership arising from a change in capital provided by the present membership which is induced by  $ds$ ; and d) the shadow price of the solution for  $s$  must equal zero.

Equations 6.32a, 6.33a, 6.34a, and 6.35a cannot be assumed to be

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<sup>1</sup>This term is also the derivation of total capital employed (Equation 6.16) where membership is held constant.

equalities. Assuming these as equalities assumes that deferred patronage dividends, capital stock, and debt are used as financial instruments and that a dividend on common stock is used as a method of allocating net savings. Rather than make these assumptions, I will state these conditions as inequalities.

Equation 6.32a has an interpretation similar to that of Equation 6.31a. We may rewrite 6.32a as;

$$\begin{aligned} \frac{\partial \Lambda}{\partial \tau} = & \frac{\partial M}{\partial \tau} \left[ -\frac{M_O}{2} \Pi_M^F + \frac{M_O}{M} \left\{ \frac{\partial T(M, K)}{\partial M} + \left[ s + \frac{(1-s)}{(1+d_m)} \tau \right] \right. \right. \\ & \left. \left[ \frac{\partial O(M, K)}{\partial M} - i P_m \right] + i P_m \right\} - \phi_1 \{ P_m + \tau(1-s) \left[ \frac{\partial O(M, K)}{\partial M} - i P_m \right] \} + \phi_2 \right] \\ & + \frac{M_O}{M} \left[ -\frac{(1-s)}{(1+d_m)} \tau \ln(1+d_m) [NS-DS] \right] \\ & - \phi_1 [(1-s)(NS-DS)] \geq 0 \end{aligned} \quad (7.14)$$

The term in the first two lines is again interpreted as the variation in the total collective profits of the present membership from changing total membership which, in Equation 7.14, is induced by  $d\tau$ .

The affects of a changing membership on the total collective profits of the present membership are analogous to the affects in the term of the first two lines of Equation 7.12. This term can be represented by  $(\partial TCP / \partial M) (\partial M / \partial \tau)$ .

The term in the third line of Equation 7.14 can be interpreted as the variation in the present value of net savings to the present membership which is induced by  $d\tau$ . This contains the derivative of the present value of net savings (Equation 6.20) with respect to  $\tau$  ( $M$  held

constant).  $[-\frac{(1-s)}{(1+d_m)^{\tau}} \ln(1+d_m)]$  is the variation in the present value factor induced by  $d\tau$  and causes the effect here. This term could be called  $\frac{M_o}{M} \frac{\partial PVNS}{\partial \tau}$ .

The term in the fourth line of Equation 7.14 can be interpreted as the variation in the total collective profits of the present membership arising from a change in total capital provided by the present membership, which is induced by  $d\tau$ . The portion of the term inside the brackets [ ] is the derivative of total capital provided by deferred patronage dividends (Equation 6.14) with respect to  $\tau$ .<sup>1</sup> The term could be called  $\frac{\partial TCP}{\partial K} \frac{\partial K}{\partial \tau}$ .

Given the interpretations of the terms in Equation 7.14, we can rewrite the equation as;

$$\frac{\partial \Lambda}{\partial \tau} = \frac{\partial TCP}{\partial M} \frac{\partial M}{\partial \tau} + \frac{M_o}{M} \frac{\partial PVNS}{\partial \tau} + \frac{\partial TCP}{\partial K} \frac{\partial K}{\partial \tau} \geq 0. \quad (7.15)$$

This equation is equivalent to stating that, for a maximum, the sum of: a) the variation in the total collective profits of the present membership from changing membership which is induced by  $d\tau$ ; b) the variation in the present value of the net savings to the present membership which is induced by  $d\tau$ ; and c) the variation in the total collective profits of the present membership arising from a change in capital provided by the present membership which is induced by  $d\tau$  must be greater than or equal to zero.

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<sup>1</sup>This can also be interpreted as the variation in total capital provided by the present membership which is induced by  $d\tau$ . This would be the derivative of Equation 6.16 with membership held constant.

Equation 6.33a can be rewritten as;

$$\frac{\partial \Lambda}{\partial D} = \frac{M_o}{M} \left\{ \left[ -r - \frac{D}{K} \frac{\partial r}{\partial D/K} \right] \left[ s + \frac{(1-s)}{(1+d_m)} \tau \right] \right\} \\ - \phi_1 \left[ \tau(1-s) \left( -r - \frac{D}{K} \frac{\partial r}{\partial D/K} \right) + 1 \right] \geq 0. \quad (7.16)$$

The sum  $-r - \frac{D}{K} \frac{\partial r}{\partial D/K}$  is the marginal interest cost of debt (MIC) which is induced by  $dD$ . The term in the first line of Equation 7.16 can be interpreted as the variation in the present value of net savings to the present membership which is induced by  $dD$ . The portion of this term in the brackets  $\{ \}$  is the derivative of the present value of net savings (Equation 6.20) with respect to  $D$ . This term can also be interpreted as the present value of marginal interest cost to the present membership. We can represent this term as  $\frac{M_o}{M} \frac{\partial PVNS}{\partial D}$ .

The term in the second line of Equation 7.16 can be interpreted as the variation in the total collective profits of the present membership arising from a change in total capital provided by the present membership, which is induced by  $dD$ .  $\tau(1-s)MIC$  is the effect debt has on the amount of capital employed by deferred patronage dividends. Debt affects patronage dividends through the amount of net savings available for distribution. Debt also affects capital by the direct addition of it to the capital employed, hence the  $+1$  in the brackets. The term in the second line could also be interpreted as the marginal profit of debt to the present membership. We can represent this term as  $\frac{\partial TCP}{\partial K} \frac{\partial K}{\partial D}$ .

Given the interpretations of the terms in Equation 7.16, we can

rewrite the equation as;

$$\frac{\partial \Lambda}{\partial D} = \frac{M_O}{M} \frac{\partial PVNS}{\partial D} + \frac{\partial TCP}{\partial K} \frac{\partial K}{\partial D} \geq 0. \quad (7.17)$$

This equation is equivalent to stating that, for a maximum, the sum of: a) the present value of marginal interest cost of debt to the present membership; and b) the marginal profit of debt to the present membership which is induced by  $dD$  must be greater than or equal to zero. We can also interpret these results to mean that the cooperative decision-maker should employ debt to the point where the marginal profit of the debt to the present membership equals, or just exceeds, the present value of the marginal interest cost to the present membership.

Equation 6.34a can be rewritten as;

$$\begin{aligned} \frac{\partial \Lambda}{\partial P_m} = & \frac{\partial M}{\partial P_m} \left[ -\frac{M_O}{M^2} \Pi_M^F + \frac{M_O}{M} \left\{ \frac{\partial T(M, K)}{\partial M} + \left[ s + \frac{(1-s)}{(1+d_m)^T} \right] \left[ \frac{\partial O(M, K)}{\partial M} - iP_m \right] \right. \right. \\ & \left. \left. + iP_m \right\} - \phi_1 \{ P_m + \tau(1-s) \left[ \frac{\partial O(M, K)}{\partial M} - iP_m \right] \} + \phi_2 \right] \\ & + \frac{M_O}{M} \left\{ \left[ s + \frac{(1-s)}{(1+d_m)^T} \right] [-iM] + iM \right\} \\ & - \phi_1 \{ M + \tau(1-s) [-iM] \} \geq 0. \end{aligned} \quad (7.18)$$

The term in the first two lines of Equation 7.18 can again be interpreted as the variation in the total collective profits of the present membership from changing membership which, in Equation 7.18, is induced by  $dP_m$ . Again, the effects of changing membership are analogous to those effects described in Equation 7.12. This term can be represented by

$$\frac{\partial TCP}{\partial M} \frac{\partial M}{\partial P_m}.$$

The term in the third line of Equation 7.18 can be interpreted as the variation in the present value of net savings to the present membership which is induced by  $dP_m$ . The portion of the term inside the brackets { } is the derivative of the present value of net savings with respect to  $P_m$  ( $M$  held constant). This term could be represented by  $\frac{M}{M} \frac{\partial PVNS}{\partial P_m}$ .

The term in the fourth line of Equation 7.18 can be interpreted as the variation in the total collective profits of the present membership arising from a change in the amount of capital provided by the present membership, which is induced by  $dP_m$ . The portion of the term inside the brackets { } is the derivation of total capital provided by the present membership (Equation 6.16) with respect to  $P_m$  with membership held constant. We can represent this term as  $\frac{\partial TCP}{\partial K} \frac{\partial K}{\partial P_m}$ .

Given the interpretations of Equation 7.18, we can rewrite the equation as;

$$\frac{\partial \Lambda}{\partial P_m} = \frac{\partial TCP}{\partial M} \frac{\partial M}{\partial P_m} + \frac{M}{M} \frac{\partial PVNS}{\partial P_m} + \frac{\partial TCP}{\partial K} \frac{\partial K}{\partial P_m} \geq 0. \quad (7.19)$$

This equation is equivalent to stating that, for a maximum, the sum of: a) the variation in the total collective profits of the present membership from changing membership which is induced by  $dP_m$ ; b) the variation in the present value of net savings to the present membership which is induced by  $dP_m$ ; and c) the variation in the total collective profits of the present membership arising from a change in the amount

of capital provided by the present membership which is induced by  $dP_m$  must be greater than or equal to zero.

Equation 6.35a has a meaning similar to that of Equation 7.19.

Equation 6.35a can be rewritten as;

$$\begin{aligned}
 \frac{\partial A}{\partial i} = & \frac{\partial M}{\partial i} \left[ -\frac{M_O}{M^2} \Pi_M^F + \frac{M_O}{M} \left\{ \frac{\partial T(M, K)}{\partial M} + \left[ s + \frac{(1-s)}{(1+d_m)^\tau} \right] \left[ \frac{\partial O(M, K)}{\partial M} - iP_m \right] \right. \right. \\
 & \left. \left. + iP_m \right\} - \phi_1 \left\{ P_m + \tau(1-s) \left[ \frac{\partial O(M, K)}{\partial M} - iP_m \right] \right\} + \phi_2 \right] \\
 & + \frac{M_O}{M} \left\{ s + \frac{(1-s)}{(1+d_m)^\tau} \right\} [-P_m M] + P_m M \} \\
 & - \phi_1 \left\{ \tau(1-s) [-P_m M] \right\} \\
 & - \phi_5 \geq 0.
 \end{aligned} \tag{7.20}$$

The first two lines of Equation 7.20 can again be interpreted as the variation in the total collective profits of the present membership from changing membership which, in Equation 7.20, is induced by  $di$ . This term could be represented by  $\frac{\partial TCP}{\partial M} \frac{\partial M}{\partial i}$ . The term in the third line of Equation 7.20 can be interpreted as the variation in the present value of net savings to the present membership which is induced by  $di$ , and can be represented by  $\frac{M_O}{M} \frac{\partial PVNS}{\partial i}$ . The term in the fourth line can be interpreted as the variation in the total collective profits of the present membership arising from a change in the amount of capital employed by the present membership, which is induced by  $di$ , and can be represented as  $\frac{\partial TCP}{\partial K} \frac{\partial K}{\partial i}$ . The final term,  $-\phi_5$ , is the Lagrange multiplier which indicates how the value of the objective function will change given a one unit change in the maximum allowed value of  $i$  (Equation 7.5).



This would be the shadow price of  $i_{\max}$  (.08) and can be represented by  $SP_i$ .

Given the interpretations of Equation 7.20, the equation can be rewritten as;

$$\frac{\partial \Lambda}{\partial i} = \frac{\partial TCP}{\partial M} \frac{\partial M}{\partial i} + \frac{M_o}{M} \frac{\partial PVNS}{\partial i} + \frac{\partial TCP}{\partial K} \frac{\partial K}{\partial i} + SP_i \geq 0. \quad (7.21)$$

This equation is equivalent to stating that, for a maximum, the sum of: a) the variation in the total collective profits of the present membership from changing membership which is induced by  $di$ ; b) the variation in the present value of net savings to the present membership which is induced by  $di$ ; c) the variation in the total collective profits of the present membership arising from a change in the amount of capital provided by the present membership which is induced by  $di$ ; and d) the shadow price of  $i_{\max}$  must be greater than or equal to zero.

Equations 6.36a through 6.40a are just restatements of the constraints imposed on the Lagrange function. These constraints were discussed in Equations 6.16, 6.25, 6.26, 6.27, and 6.28.

#### The Fixed Membership and Capital Model

Most authors have assumed fixed membership and fixed amount of capital when analyzing cooperative financial structure. The two most common methods of analyzing capital structure have been minimizing the cost of capital employed in the cooperative association and maximizing

the returns to the member patrons in a simulation analysis. Both of these methods assume a fixed membership and often assume fixed capital employment levels.

Similar assumptions can be made with the model in the previous section. In the model to be presented now, the cooperative decision-maker will be concerned with maximizing the total collective profits of the member patrons assuming the capital employment and membership level are fixed. This model will allow us to compare other methods of analyzing cooperative financial structure with the model developed in the previous section. Arguments may be presented where the time frame (long-run vs. intermediate-run vs. short-run) for determining the optimum membership level and capital employment level differs with the time frame for the decision on the financial structure. The sequence of the decision would be that membership and capital is determined first, and that the composition of the capital is determined second. This, however, would violate the results perceived in the patron model in Chapter VI (Equation 6.8) where membership was determined by the decision variables in the finance model. This model would require more restrictive membership requirements than the model presented in Chapter VI.

A second argument which may be used is that membership is fixed by the organization of member patrons into a cooperative association and membership would not be allowed to change after the initial decision to join. It could then be argued that the first decision facing the cooperative decision-maker is determining the level of capital to employ, and then, secondly, determining the composition of that capital. The

assumptions for this model will put the cooperative decision-maker in this second phase of the capital structure decision.

In this model, the total revenues to the member patrons received for products in set X and set Y will be fixed because membership and capital are fixed, i.e.;

$$T(M,K) = \bar{T}. \quad (7.22)$$

In addition, net savings may be expressed as;

$$NS = \bar{O} - rD \quad (7.23)$$

where  $\bar{O}$  is the fixed level of operating income (fixed because membership and capital are fixed) and  $r$  and  $D$  are as defined before.  $DS$  will also be as defined in Equation 6.12. Given these definitions, we can express the objective function of the cooperative decision-maker as;

$$\bar{\Pi}_M^F = \bar{T} + [s + \frac{(1-s)}{(1+d_m)^\tau}] [NS-DS] + DS. \quad (7.24)$$

The  $\bar{\Pi}_M^F$  term is the total collective profits of the present membership with fixed capital employment and membership. The bar over the  $F$  represents fixed capital employment and the bar over the  $M$  represents fixed membership.

The operation of the cooperative will be subject to four constraints. The first constraint will fix total capital at the predetermined level,  $\bar{K}$ , i.e.;

$$\bar{K} - p_m \bar{M} - \tau(1-s) (NS-DS) - D = 0. \quad (7.25)$$

The second, third, and fourth constraints are the same as Equations 6.26, 6.27, and 6.28. We can state the Lagrangian expression as;

$$\begin{aligned} \bar{\Lambda}_M^F = & \bar{I} + [s + \frac{(1-s)}{(1+d_m)\tau}] [NS-DS] + DS + \xi_1 [\bar{K} - P_m \bar{M}] \\ & - \tau(1-s)(NS-DS)-D] + \xi_2 [s-.2] + \xi_3 [1-s] + \xi_4 [.08-i]. \end{aligned} \quad (7.26)$$

The decision variables available to the cooperative decision-maker include the same variables as in the previous model except for  $K$ : the price of a share of common stock ( $P_m$ ); the proportion of patronage dividends paid in cash ( $s$ ); the length of the revolving fund ( $\tau$ ); total debt employed ( $D$ ); and the dividend rate on total common stock investment ( $i$ ). The Kuhn-Tucker conditions follow. The first five can be obtained from 6.31 through 6.35 by setting all derivations of  $M$  equal to zero.

$$\frac{\partial \bar{\Lambda}_M^F}{\partial P_m} = [s + \frac{(1-s)}{(1+d_m)\tau}] [-i\bar{M}] + i\bar{M} + \xi_1 [-\bar{M} + \tau(1-s)(i\bar{M})] \geq 0 \quad (7.27a)$$

$$\frac{\partial \bar{\Lambda}_M^F}{\partial P_m} \cdot P_m = 0 \quad (7.27b)$$

$$P_m \geq 0 \quad (7.27c)$$

$$\frac{\partial \Lambda_{\overline{M}}^{\overline{F}}}{\partial s} = [1 - \frac{1}{(1+d_m)^\tau}] [\text{NS-DS}] + \xi_1 [\tau (\text{NS-DS})] + \xi_2 - \xi_3 \geq 0 \quad (7.28a)$$

$$\frac{\partial \Lambda_{\overline{M}}^{\overline{F}}}{\partial s} \cdot s = 0 \quad (7.28b)$$

$$s \geq 0 \quad (7.28c)$$

$$\frac{\partial \Lambda_{\overline{M}}^{\overline{F}}}{\partial \tau} = [-\frac{(1-s)}{(1+d_m)^\tau} \ln(1+d_m)] [\text{NS-DS}] - \xi_1 [(1-s) (\text{NS-DS})] \geq 0 \quad (7.29a)$$

$$\frac{\partial \Lambda_{\overline{M}}^{\overline{F}}}{\partial \tau} \cdot \tau = 0 \quad (7.29b)$$

$$\tau \geq 0 \quad (7.29c)$$

$$\begin{aligned} \frac{\partial \Lambda_{\overline{M}}^{\overline{F}}}{\partial D} &= [s + \frac{(1-s)}{(1+d_m)^\tau}] [-r - \frac{D}{K} \frac{\partial r}{\partial D/K}] \\ &- \xi_1 [\tau (1-s) (-r - \frac{D}{K} \frac{\partial r}{\partial D/K}) + 1] \geq 0 \end{aligned} \quad (7.30a)$$

$$\frac{\partial \Lambda_{\overline{M}}^{\overline{F}}}{\partial D} \cdot D = 0 \quad (7.30b)$$

$$D \geq 0 \quad (7.30c)$$

$$\frac{\partial \Lambda_{\overline{M}}^{\overline{F}}}{\partial i} = [s + \frac{(1-s)}{(1+d_m)^\tau}] [-p_{\overline{m}} \overline{M}] + p_{\overline{m}} \overline{M} - \xi_1 [\tau (1-s) (-p_{\overline{m}} \overline{M})] - \xi_4 \geq 0 \quad (7.31a)$$

$$\frac{\partial \Lambda_{\overline{M}}^{\overline{F}}}{\partial i} \cdot i = 0 \quad (7.31b)$$

$$i \geq 0 \quad (7.31c)$$

$$\frac{\partial \Lambda_{\overline{M}}}{\partial \xi_1} = \overline{K} - P_m \overline{M} - \tau(1-s)(NS-DS) - D = 0 \quad (7.32)$$

$$\frac{\partial \Lambda_{\overline{M}}}{\partial \xi_2} = s - .2 \geq 0 \quad (7.33a)$$

$$\frac{\partial \Lambda_{\overline{M}}}{\partial \xi_2} \cdot \xi_2 = 0 \quad (7.33b)$$

$$\xi_2 \geq 0 \quad (7.33c)$$

$$\frac{\partial \Lambda_{\overline{M}}}{\partial \xi_3} = 1.0 - s \geq 0 \quad (7.34a)$$

$$\frac{\partial \Lambda_{\overline{M}}}{\partial \xi_3} \cdot \xi_3 = 0 \quad (7.34b)$$

$$\xi_3 \geq 0 \quad (7.34c)$$

$$\frac{\partial \Lambda_{\overline{M}}}{\partial \xi_4} = .08 - i \geq 0 \quad (7.35a)$$

$$\frac{\partial \Lambda_{\overline{M}}}{\partial \xi_4} \cdot \xi_4 = 0 \quad (7.35b)$$

$$\xi_4 \geq 0 \quad (7.35c)$$

The condition a of Equations 7.27 through 7.31 may be rearranged as;

$$\frac{\partial \Lambda_{\overline{M}}}{\partial P_m} = - \overline{M} \left\{ s + \frac{(1-s)}{(1+d_m)^{\tau}} - 1 - \xi_1 \tau(1-s) \right\} i + \xi_1 \geq 0 \quad (7.27a')$$

$$\frac{\partial \Lambda_{\overline{M}}^{\overline{F}}}{\partial s} = [\text{NS-DS}] \left[ 1 - \frac{1}{(1+d_m)^\tau} + \xi_1 \tau \right] + \xi_2 - \xi_3 \geq 0 \quad (7.28a')$$

$$\frac{\partial \Lambda_{\overline{M}}^{\overline{F}}}{\partial \tau} = [\text{NS-DS}] \left[ -\frac{(1-s)}{(1+d_m)^\tau} \ln(1+d_m) - \xi_1 (1-s) \right] \geq 0 \quad (7.29a')$$

$$\frac{\partial \Lambda_{\overline{M}}^{\overline{F}}}{\partial D} = \left[ -r - \frac{D}{K} \frac{\partial r}{\partial D/K} \right] \left[ s + \frac{(1-s)}{(1+d_m)^\tau} - \xi_1 \tau (1-s) \right] - \xi_1 \geq 0 \quad (7.30a')$$

$$\frac{\partial \Lambda_{\overline{M}}^{\overline{F}}}{\partial i} = -P_m \overline{M} \left[ s + \frac{(1-s)}{(1+d_m)^\tau} - 1 - \xi_1 \tau (1-s) \right] - \xi_4 \geq 0. \quad (7.31a')$$

The Lagrange multipliers,  $\xi_1$  through  $\xi_4$ , could be interpreted in the same way as the Lagrange multipliers in the general model with the corresponding constraints.

Equation 7.33a guarantees that  $s$  is greater than zero, i.e., Equation 7.28a' must be satisfied as an equality. Beyond this, assumptions of the relationships would result in assumptions about the results of these equations. No definite conclusions can be drawn about the results without making additional assumptions. If we do assume deferred patronage dividends are used as a method of financing the cooperative association, then the length of the revolving fund ( $\tau$ ) will be positive and the cash portion of the patronage dividends ( $s$ ) will be less than one.<sup>1</sup> In addition, we will assume NS-DS is positive, the solution for  $s$  is not on the boundary (i.e.,  $\xi_2$  and  $\xi_3$  equal zero),

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<sup>1</sup>For the cooperative to use deferred patronage dividends, it must defer a portion of the dividends ( $1-s$ , i.e.,  $s < 1.0$ ) for a positive number of years ( $\tau > 0$ ).

and  $1+d_m$  is greater than one.<sup>1</sup> Given these assumptions, we can determine from Equation 7.29a' that;

$$\xi_1 = - \frac{\ln(1+d_m)}{(1+d_m)^\tau} . \quad (7.36)$$

From Equation 7.28a we can determine that;

$$1 - \frac{1}{(1+d_m)^\tau} + \xi_1 \tau = 0. \quad (7.37)$$

Substituting the right hand side of Equation 7.36 into Equation 7.37 for  $\xi_1$ , we find

$$1 - \frac{1}{(1+d_m)^\tau} - \frac{\tau \cdot \ln(1+d_m)}{(1+d_m)^\tau} = 0. \quad (7.38)$$

The only solution for  $\tau$  in Equation 7.38 is zero. This implies that deferred patronage dividends are not used as a capital source, which violates the assumption made in order to solve the equation. If the expression on the left hand side of Equation 7.38 was positive, then a positive solution to  $\tau$  could be determined. This result could be obtained if the solution for  $s$  was .2 and the shadow price of  $s_{\min}(\xi_2)$  was negative. There is, however, no reason given in Equations 7.27a' through 7.31a' to expect the result of  $s$  equal to .2 and  $\xi_2$  to be positive.

Given the result of  $\tau$  equal to zero, the solution for  $s$  is irrelevant, since the deferred portion of patronage dividends is paid in cash. We could, therefore, express the solution for  $s$  as equal to 1.0 since,

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<sup>1</sup> $d_m$  is the average discount rate of the membership. We would expect  $d_m > 0$  since the opportunity cost of money should be positive.



when  $\tau$  equals zero we are actually paying out deferred patronage dividends at the same time we pay the cash portion ( $s$ ) of patronage dividends.

Multiplying Equation 7.37 by  $-(1-s)$ , we find;

$$s + \frac{(1-s)}{(1+d_m)^\tau} - 1 - \xi_1 \tau (1-s) = 0. \quad (7.39)$$

Given this equation, we can determine from Equations 7.30a' and 7.39 that;

$$[-r - \frac{D}{K} \frac{\partial r}{\partial D/K}] \geq \xi_1. \quad (7.40)$$

The term  $[-r - \frac{D}{K} \frac{\partial r}{\partial D/K}]$  is the marginal interest cost of debt (MIC).

If we assume a positive solution for debt, then from Equations 7.36 and 7.40 we find that;

$$\text{MIC} = - \frac{\ln(1+d_m)}{(1+d_m)^\tau}, \quad (7.41)$$

and, since  $\tau=0$ ;

$$\text{MIC} = -\ln(1+d_m). \quad (7.42)$$

The solution of Equation 7.42 implies that the cooperative decision-maker should employ debt to the point where the marginal interest cost is less than the negative of the average discount rate for the membership. This result is determined since the first derivative of the right hand side of Equation 7.42 is negative  $(-\frac{1}{1+d_m})$  with respect to  $d_m$  and the second derivative is positive,  $(-\frac{1}{(1+d_m)^2})$ . MIC equals  $d_m$  when  $d_m$  is zero and MIC is less in absolute value than is  $d_m$  when  $d_m$  is positive.

The solution for  $P_m$  can be found from Equation 7.32, i.e.;

$$\bar{K} - P_m \bar{M} - D^* = 0 \quad (7.43)$$

or;

$$P_m = \frac{\bar{K} - D^*}{\bar{M}} \quad (7.44)$$

where  $D^*$  is the solution for  $D$  determined in Equation 7.42.  $\tau$  was set equal to zero to yield Equation 7.43.

The solution for  $i$  is irrelevant because  $\tau$  equal to zero means that paying dividends on common stock is equivalent to paying cash patronage dividends. Since Equation 7.39 was expressed as an identity we can rewrite 7.31a' as;

$$-\xi_4 \geq 0. \quad (7.45)$$

From Equation 7.45 and Equation 7.35c we find that;

$$\xi_4 = 0. \quad (7.45)$$

We can interpret this as meaning that the shadow price of the maximum value of  $i$  is equal to zero, which indicates only that the solution for  $i$  is between 0 and .08. No further results can be obtained from the Kuhn-Tucker conditions about  $i$ .

The system of Equations 7.27a' through 7.31a' dictate specific solutions for  $\tau$ ,  $D$ , and  $P_m$ . Given fixed membership and capital indicates that the burden of financing the cooperative association should rest on debt and common stock investment. The result of this model does indicate a result different from most other models constructed for analyzing financial structure. Equation 7.42 indicates that the marginal interest cost of debt should be less in absolute value than

the average discount rate for the membership. The normal solution indicates that MIC and  $\frac{1}{1+d_m}$  should be equal. The results from empirical studies using this latter criterion showed that cooperatives often rely too heavily on member financing and not enough on debt to finance the cooperative. The result of the simplified model constructed in this chapter appears to come closer to fitting reality in the determination of the amount of debt to use. The other major difference in this model from other models constructed to study financial structure lies in the fact that only common stock investment and debt are used in financing the cooperative. Many other authors who have analyzed cooperative financial structure with fixed membership have also assumed a fixed common stock price ( $P_m$ ). This assumption leads to the result that deferred patronage dividends should be used to satisfy the capital obligation left after determining the level of debt to employ, i.e.;

$$DPD = \bar{K} - \bar{P}_m \bar{M} - D^* \quad (7.43)$$

where DPD is the amount of deferred patronage dividends required,  $P_m$  is the fixed common stock price and  $D^*$  is the level of debt employed (where  $MIC = \frac{1}{1+d_m}$ ). In this model constructed with variable common stock price,  $\tau$  was determined to equal zero, i.e., DPD would also equal zero and all member financing was obtained via common stock investment. The determining factor of whether deferred patronage dividends are used is the price of common stock. If common stock price is fixed, then member investment does come from deferred patronage dividends. If common stock price is not fixed then no deferred patronage dividends are

used with fixed membership and capital employment.

### The Fixed Membership With Variable Capital Model

Variable capital allows the cooperative decision-maker to choose the total amount of capital to employ that will give the member patrons the maximum total collective profits. In this model, we would change the objective function since total revenues for products in sets X and Y are dependent on capital and the operating income portion of net savings is dependent on capital, i.e., we derive the following Lagrangian function;<sup>1</sup>

$$\begin{aligned} \Lambda_{\bar{M}}^F = & T(K) + [s + \frac{(1-s)}{(1+d_m)^{\tau}}] [NS-DS] + DS + a_1 [K-P_m \bar{M}] \\ & - \tau(1-s)(NS-DS) - D] + a_2 [s - .2] + a_3 [1-s] \\ & + a_4 [.08-i], \end{aligned} \quad (7.44)$$

where NS is the net savings defined as;

$$NS = O(K) - rD. \quad (7.45)$$

The decision-maker faces the same decision variables as in the fixed membership and capital model with the addition of capital (K) as a variable. The Kuhn-Tucker conditions for this model will be the same as for the fixed membership and capital model where the Lagrange multipliers are the same, capital is not fixed, and in this model we have an additional condition for capital, i.e.;

<sup>1</sup>The Lagrangian function is called  $\Lambda_{\bar{M}}^F$  since we have variable capital (no bar over the F) and fixed membership (bar over the M).

$$\begin{aligned} \frac{\partial \Lambda_M^F}{\partial K} = \frac{\partial T(K)}{\partial K} + \left[ s + \frac{(1-s)}{(1+d_m)^\tau} \right] \left[ \frac{\partial O(K)}{\partial K} + \frac{D^2}{K^2} \frac{\partial r}{\partial D/K} \right] \\ + a_1 \left[ 1 - \tau(1-s) \left( \frac{\partial O(K)}{\partial K} + \frac{D^2}{K^2} \frac{\partial r}{\partial D/K} \right) \right] = 0. \end{aligned} \quad (7.46)$$

The term,  $\frac{\partial O(K)}{\partial K} + \frac{D^2}{K^2} \frac{\partial r}{\partial D/K}$ , is the marginal net savings induced by  $dK$  ( $MNS_K$ ) and 7.46 may be rewritten as;

$$\frac{\partial \Lambda_M^F}{\partial K} = \frac{\partial T(K)}{\partial K} + \left[ s + \frac{(1-s)}{(1+d_m)^\tau} - a_1 \tau(1-s) \right] MNS_K + a_1 = 0 \quad (7.47)$$

If we assume all Lagrange multipliers with the exception of  $a_1$  equal zero, then we derive the same results as in the fixed membership and capital model, with the additional result;

$$\frac{\partial T(K)}{\partial K} + MNS_K = \ln(1+d_m) \quad (7.48)$$

Equation 7.48 is derived from Equations 7.36 ( $a_1$  equivalent to  $\xi_1$ ), 7.39, and 7.47. The equation is equivalent to stating that capital should be employed to the point where the variation in profit to the membership,  $\frac{\partial T(K)}{\partial K} + MNS_K$ , is exactly equal to the natural logarithm of  $(1+d_m)$  which would also equal the negative of MIC.

Assuming a variable capital level allows an optimal solution for capital employment which guarantees that capital is employed at the point where its variation in profit to the membership equals the natural logarithm of  $(1+d_m)$ . This adds the additional solution of determining the amount of capital to employ in the cooperative association. From this, the level of debt to employ and the price of common stock may be found from Equations 7.42 and 7.44.

# The Variable Membership and Capital Model

Eliminating the final assumption of fixed membership makes the model identical to the general model developed earlier. In this model we derive the Lagrangian expression defined in Equation 6.29. Membership was shown to vary with the decision variables in the financial model as shown in Chapter VI (Equation 6.8). The Kuhn-Tucker conditions were expressed in Equations 6.30 through 6.40 and discussed earlier in this chapter. The major difference in the Kuhn-Tucker conditions for the general model and those for the fixed membership model is the expression for the effects of changing membership ( $\frac{\partial M}{\partial j}$ , where  $j$  is one of the decision variables) and the normalizing constant for present membership ( $\frac{M_0}{M}$ ).

The results of the models with fixed membership, or fixed membership and capital, determined values for the decision variables  $K$ ,  $D$ ,  $P_m$ , and  $\tau$ .  $s$  and  $i$  were determined to be irrelevant in the solution. In the general model the effect of a variable membership will be to dictate specific solutions for all of the decision variables, since each decision variable  $s$ ,  $\tau$ ,  $P_m$ , and  $i$  will have an effect on the level of membership.

The Kuhn-Tucker conditions 6.30 through 6.40 are very complex. A great amount of information is needed to evaluate them. The cooperative decision-maker must know the specific form of the membership function, the response function of total sales revenues for products in sets  $X$  and  $Y$  to capital and membership, the response function

function of operating income to capital and membership, and the form of the interest rate function. In addition, the cooperative decision-maker must know the average cost of capital for the membership ( $d_m$ ).

The optimality conditions presented here do provide some value to the cooperative who is attempting to maximize the total collective profits of the present membership. The conditions also point out the difficult task cooperative decision-makers have in fulfilling this task. They also point to the need for research to provide an understanding of these relationships.

## CHAPTER VI.II. SUMMARY AND CONCLUSIONS

## Summary and Conclusions

Some of the problems associated with cooperative associations were discussed, and the relevant literature was reviewed to see how well these problems have been studied and solutions provided. The purpose of this study was to discuss the overall decision nexus of the cooperative decision-maker and analyze the separate problems involved in this decision nexus.

The decision nexus of the cooperative was separated into three interdependent problems. The first problem discussed was the short-run determination of pricing and production practices. The second problem was the long-run problem of determining an investment portfolio for the cooperative. This problem was not discussed at length. The third problem was the long-run problem of determining the financial structure of the cooperative association. The study of the decision nexus was important in that it did specify the link between the short-run production and pricing decision problem and the long-run asset and financial structure decision problems. The long-run decision problems prescribe the structure the cooperative decision-maker must operate within the short-run. The long-run decision problem encompasses a larger number of variables which can be controlled by the cooperative decision-maker.

The individual problems of determining the production and pricing practices within cooperatives and determining the financial structure of cooperatives were discussed at length. The study of the production and pricing practices within cooperatives was a positive-descriptive study



of how cooperatives actually determine prices for products they trade with patrons. The normative model developed by Royer was used as a basis for studying pricing practices. From a survey of cooperatives in Iowa, models were statistically defined using ordinary least squares for the feed and fertilizer models and generalized least squares for the corn and soybean models. It was discovered that characteristics of cooperatives could be used to describe part of the variance in the prices for products across cooperatives. The feed and fertilizer models used information from sixty-eight Iowa cooperatives to develop pricing models for anhydrous ammonia fertilizer and a 40 percent hog concentrate feed ratio. The anhydrous ammonia model found that plant storage capacity in tons of anhydrous ammonia, the service charge in dollars for delivery and application of anhydrous ammonia to the patron, the percentage of grain production which is corn in the county of the firm's home office, and the amount of capital in millions of dollars employed by the cooperative all have a negative effect (i.e., the higher is the value of the variable the lower is the price of fertilizer) on the price of anhydrous ammonia. The equipment storage capacity in tons and leverage ratio of the firm (debt/total liabilities) had a positive effect on the price of anhydrous ammonia.

The model estimated for the 40 percent hog ration found that the leverage ratio and number of laying hens in thousands in the county of the firm's home office had a positive effect on the price of the hog ration whereas the amount of capital employed by the cooperative had a negative effect on the price of the hog ration.

Models for 35 and 50 percent cattle feed rations were unable to be estimated. Data were collected concerning these variables but measurement error appeared to make it impossible to fit a suitable model.

Corn and soybean prices were collected from the cooperatives in the form of a weekly time series running from January, 1977, to June, 1979. The Chicago Board of Trade price for each day was subtracted from the cooperative price to get the basis level as the dependent variable. A generalized least squares regression analysis was used because the data consisted of time series and cross-sectional information. Two models were estimated for each of the corn and soybean price data sets. One model consisted of the time series variables trend, monthly seasonal dummy variables, and a trigonometric seasonal term which allowed for multiplicative seasonal effects. The second model contained the same variables plus a storage utilization variable. In addition to these time series variables, twenty-two cross-sectional variables were used in the regression analysis. The models allowed for effects from the time series variables, effects from the cross-sectional variables, and effects from the interaction of the cross-sectional and time series variables.

Of the twenty-two cross-sectional variables used in the regression analysis, no one variable consistently had a positive or negative effect on the price of either grain. The different months of the year either produced different effects from the cross-sectional variables, or else the cross-sectional variable was not consistently significant.

The study of the financial structure of cooperatives was a normative-

prescriptive study of a multi-product marketing and supply cooperative which served both member and nonmember patrons. This part of the study involved the development of a model and then defining the economic implications of the model. Development of the model began with the construction of a model for a typical patron for determination of whether to become a member. The typical patron was assumed to be a profit maximizer who purchased inputs from and sold unprocessed products to the cooperative association.

From the optimality conditions determined for this model, a membership function was derived for the cooperative association. The prices for products set inside and outside the cooperative association, the quantity of public goods produced by the cooperative association, the proportion of patronage dividends paid in cash, the number of years deferred patronage dividends are deferred, the discount rate of the member, the expected dividend rates of the member, the dividends on common stock, and the price of a share of common stock are the variables which determine the level of membership within the cooperative association.

The cooperative decision-maker was assumed to maximize the total collective profits of the present membership. The optimality conditions for the general model of the cooperative were analyzed. Simplified models, including that where membership and capital were fixed, and that where just membership was fixed, were also analyzed and the results were compared to the results obtained in most of the literature.

In the model where membership and capital were fixed it was discovered that the cooperative should employ debt to a point where the

marginal interest cost of debt is less than the negative of the average discount rate for the membership. These results differ from the results derived in most other models which assume a fixed membership level and fixed capital level. Most other studies which make these assumptions state that the cooperative should employ debt to the point where the marginal interest cost equals the negative of the average discount rate of the membership. The empirical results of most financial structure studies have found that cooperatives generally have not used debt up to the point where the marginal interest cost equals the negative of the average discount rate of the membership, but that the marginal interest cost is in fact less than the average discount rate. These empirical studies would support the hypothesis that cooperative decision-makers use the criteria for determining the financial structure as developed in the model presented in this text.

Adding capital as a variable in the model did not substantially change the results from those derived in the previous model which had a fixed membership level and a fixed capital level. It did allow the cooperative to determine the level of capital to employ where the variation in profit to the membership is less than the average discount rate of the membership.

Adding the final assumption that the level of membership is variable along with the amount of capital being variable gives the general model that was first discussed. The main conclusion derived from the general model is that with a variable membership, the effect of the choice variables on membership will alter the results of the simplified

model. If the form of the membership function, total sales revenue function, and operating income function are known, the specific solutions for each of these variables should exist.

The principal conclusion determined in this study is that the task of the cooperative decision-maker is difficult. The cooperative decision-maker must be concerned about the short-run and long-run decisions which affect the economic positions of the cooperative and its member patrons. The optimality conditions derived for the short-run production and pricing model point to the complexity of maximizing the member patrons' total collective profits.

The optimality conditions derived for the long-run model for determining the financial structure of the cooperative also point to the difficulty of the task of the cooperative decision-maker. Not only are the optimality conditions complex, but there is a great amount of information which is necessary to evaluate them. Nevertheless, many of the results obtained in this model should be useful. Although it is doubtful that any cooperative association could maximize the total collective profits of the member patrons, the results should be useful for providing simple rules which might help the cooperative decision-maker in striving for this goal.

### Suggestions for Further Research

In studying the pricing practices of the cooperatives in this study, it was pointed out that the data for the cattle feed models was inadequate to yield a model which was statistically significant. The models which were developed for anhydrous ammonia and hog feed were simple models which should be able to be improved. To do this would require a more extensive survey. The sample should be expanded to include more firms so that a more precise model could be developed.

It would also be interesting to see how cooperatives differ from proprietary corporations in the pricing of products. This would require a survey of proprietor corporations to see if the pricing mechanism differs between the two types of corporations, and by how much.

Finally, it would be interesting to try to develop a programming model of the cooperative association which could be used in a positive-descriptive analysis of cooperative financing. The model developed in this dissertation should provide an economic basis for development of the programming model.

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